S3I: MATLAB® Seismic Imaging Toolbox

An integrated solution for 3D seismic image modeling, migration, and full-waveform inversion (FWI)

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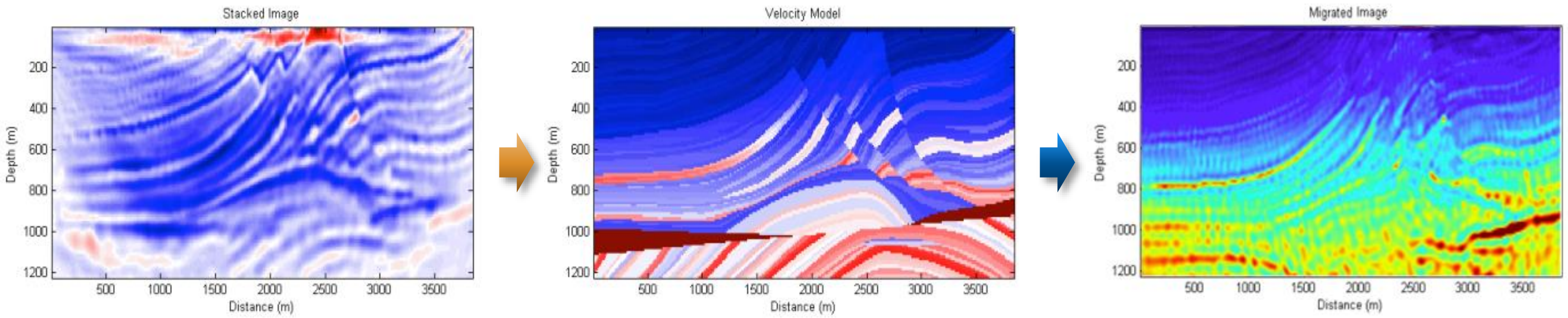


Artificial Intelligence | Big Data Analysis | Deep Learning | Machine Learning | Reinforced Learning | Predictive Analytics | Internet of Things | Process Optimization | Process Digitization | Process Automation | Value Chain Integration

MathWorks

Accelerating the pace of engineering and science

Outline

- MathWorks® digital solutions
- S3I: MATLAB® Seismic Imaging Toolbox
  - S3I in a nutshell
  - S3I highlights
  - S3I examples
  - S3I resources

MathWorks Digital Solutions
Fact Sheet Highlights

- MathWorks® is a private company founded in Massachusetts, USA in 1984 to:
  - Provide the ultimate computing environment for technical computation, visualization, design, simulation, and implementation
  - Accelerate the pace of discovery, innovation, development, and learning in engineering and science
- MathWorks® has developed major digital solutions for industry and academia:
  - MATLAB®, the language of engineers and scientists, for algorithm development, data analysis, visualization, and numerical computation.
  - Simulink®, a block diagram environment for model-based design and simulation of multidomain and embedded engineering systems; plus…
  - …over 120 digital products for data analytics, image/signal processing, control systems, robotics, deep learning, digital twins, and many, many more.

MathWorks® in Energy Resources

Data Science Workflow for Big Data Analysis

Data Processes

- **Big-Data Analysis**
- **High-Power Computing**
- **Predictive Analytics**

Technical Requirements

Database size vs. Acquisition frequency (months → hours):
- Subsurface data (Tb-Pb)
- Monitoring data (Gb-Tb)
- Production data (Mb-Gb)

Computing capacity vs. Processing turnaround (hours → seconds):
- Parallel Computing
- GPU Computing
- Cloud Computing
- Quantum Computing

Process Complexity vs. Data Predictability (low → high):
- Image/Signal Processing
- Machine Learning
- Computer Vision
- Deep Learning

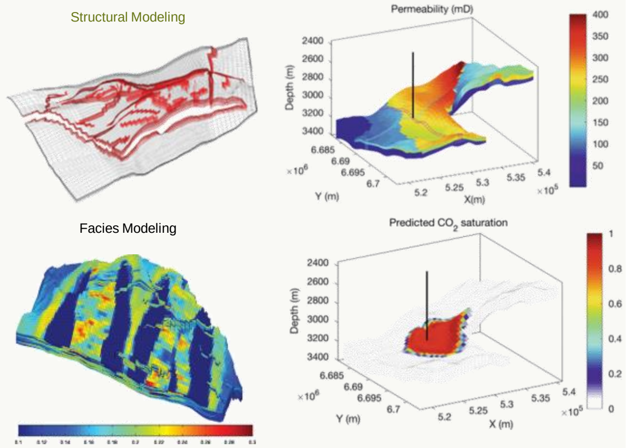
MATLAB & SIMULINK

- MATLAB® Production Server
- MATLAB® Cloud Service
- MATLAB® Quantum Computing
- Parallel Computing Toolbox
- GPU Computing Toolbox
- Simulink® Simscape
- Image/Signal Processing Toolboxes
- S3I Toolbox
- Machine Learning Toolbox
- Deep Learning Toolbox

Business Applications

UPSTREAM

Subsurface Process Modeling | Subsurface Process Simulation



DOWNSTREAM

Dynamic Process Modeling & Simulation

Simscape: Electrical, Mechanical, Magnetic, Thermal, Custom equations, Hydraulic, Thermal Liquid, Two-Phase Fluid, Gas, Moist Air

MathWorks® – Digital Subsurface Toolset (v2024)

S3I
Subsurface Imaging

SeReM
Subsurface Modeling

MRST
Subsurface Simulation

AI
Shalllow & Deep Learning

HPC
CPU | GPU | Cloud | Quantum

STEM
Digital Geoscience & Engineering Analysis

BDA & HPC
Big Data Analysis & High-Power Computing

MATLAB®

Key technology differentiators

- Customizable subsurface toolboxes fully developed and interconnected on MATLAB® platform
- Model-based and data-driven geoscience & engineering workflows to maximize data & image usage
- MathWorks® support, training, and development of data science, engineering, and analytics solutions
- Adaptive digital solutions to assess and integrate new energy processes using high-end technologies
- Low-cost, high-quality software solution to maximize technical expertise, IT infrastructure, and budget
- 200+ energy companies globally currently use MATLAB® solutions across upstream and downstream

S3I: MATLAB® Seismic Imaging Toolbox

- S3I is a MATLAB®-based toolbox developed to model, process, migrate, and invert 3D seismic images for subsurface facies characterization

- S3I was developed by Research Geoscientists from Georgia Institute of Technology to offer geophysical processing methods to unfold high-fidelity depth imaging and subsurface characterization from seismic waveforms

- S3I offers 3D modeling of acoustic and elastic wave equations, Kirchhoff, Reverse Time (RTM), and Least Squares (LSM) migrations, and full waveform inversion (FWI) in the frequency domain.

- S3I is a MATLAB® open-source toolkit available at GitHub – S3I

Key Advantages of S3I

- MATLAB®-based environment to customize and adapt seismic modeling, processing, and inversion workflows to support subsurface characterization

- Wide range of 3D data-driven and model-based solutions to enhance seismic depth imaging quality in complex and noisy environments

- Proven successful to predict subsurface facies properties from seismic

- Robust geophysical methods including full-waveform inversion (FWI)

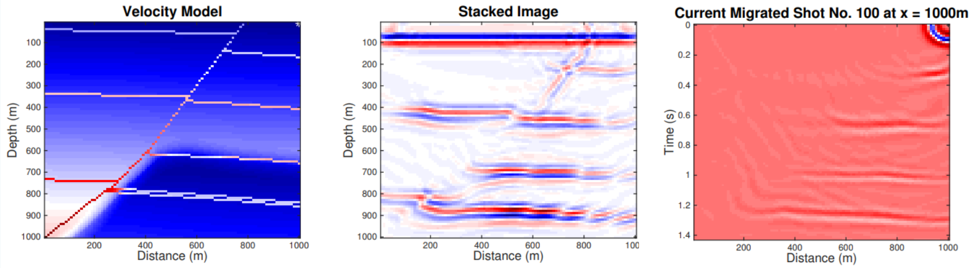
- Optimized convolution, migration, and inversion methods using MATLAB

MathWorks® in Energy Resources

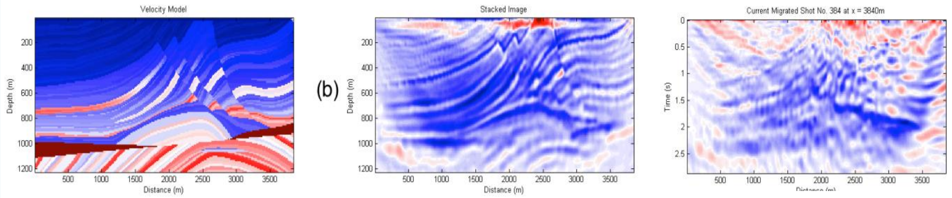
S3I: MATLAB Seismic Imaging Toolbox

S3I: 3D seismic image modeling, simulation, and migration (PreSTM, RTM, LSM, FWI)

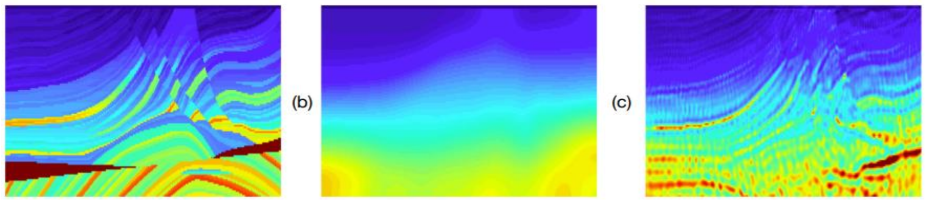
2D Seismic Full-Waveform Modeling & Migration (PreSTM)



3D Reverse Time & Least Squares Migration (RTM & LSM)



3D Seismic Full-Waveform Inversion (FWI)



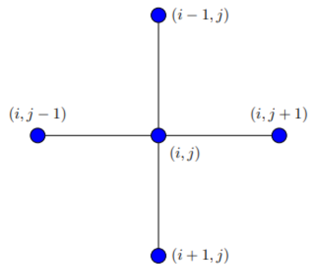
- **Seismic full-waveform modeling and simulation using finite difference method (FEM) and parallel computing capabilities**
- **Prestack Time Migration (PreSTM) based on Kirchhoff's ray-tracing, double-square-root (DSR) solution to wave equation**
- **Reverse Time Migration (RTM) to enhance depth imaging without dip limitations based on a broadband wave equation**
- **Least Squares Migration (LSM) to enhance depth imaging by reducing RTM crosstalk in complex structures.**
- **Full-Waveform Inversion (FWI) in frequency domain using a projected quasi-Newton (PQN) solution to Green's functions**

S3I highlights

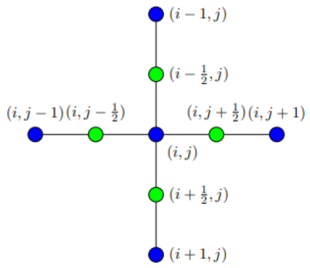
Accelerated seismic acoustic wave modeling using standard and staggered grids and multi-CPU computing protocols to reduce turnaround times on large datasets

Acoustic wave equation solutions

$$\frac{1}{v^2(\boldsymbol{x})}\frac{\partial^2 p(\boldsymbol{x},t)}{\partial t^2} + f(\boldsymbol{x},t) = \nabla^2 p(\boldsymbol{x},t)$$



(a) Standard Grid

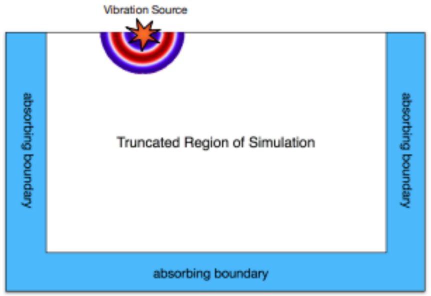


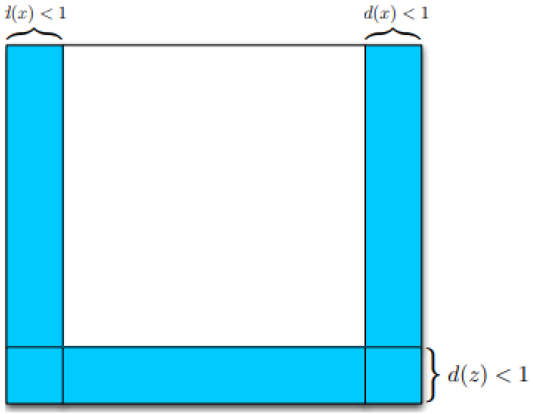
(b) Staggerd Grid

$$p_{i,j}^{(n+1)} = \frac{v_{i,j}^2\Delta t^2}{\Delta z^2}\left(p_{i+1,j}^{(n)} - 2p_{i,j}^{(n)} + p_{i-1,j}^{(n)}\right) + \frac{v_{i,j}^2\Delta t^2}{\Delta x^2}\left(p_{i,j+1}^{(n)} - 2p_{i,j}^{(n)} + p_{i,j-1}^{(n)}\right) + 2p_{i,j}^{(n)} - p_{i,j}^{(n-1)} + v_{i,j}^2\Delta t^2 f_{i,j}^{(n)}.$$

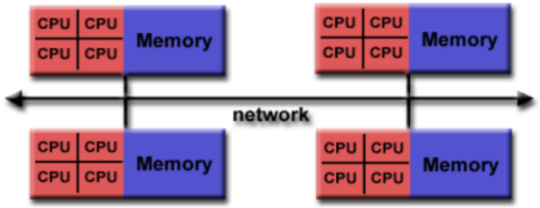
$$\frac{\partial p(u)}{\partial u} = \sum_{k=0}^{N-1} a_k \frac{p\left(u+\frac{2k+1}{2}\Delta u\right) - p\left(u-\frac{2k+1}{2}\Delta u\right)}{(2k+1)\Delta u}$$
$$= \sum_{k=0}^{N-1} a_k \left[\frac{\partial p(u)}{\partial u} + \frac{\Delta u^2}{3!\cdot 2^2}(2k+1)^2\frac{\partial^3 p(u)}{\partial u^3} + \frac{\Delta u^4}{5!\cdot 2^4}(2k+1)^4\frac{\partial^5 p(u)}{\partial u^5} + \cdots + \frac{\Delta u^{2N-2}}{(2N-1)!\cdot 2^{2N-2}}(2k+1)^{2N-2}\frac{\partial^{2N-1} p(u)}{\partial u^{2N-1}} + o(\Delta u^{2N})\right]$$

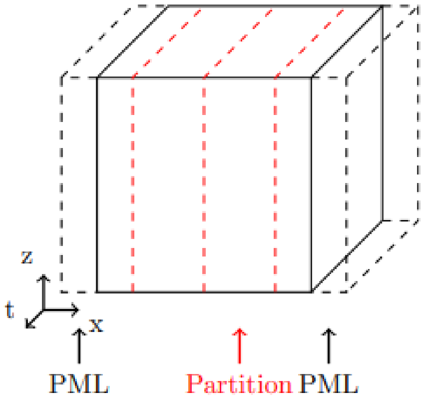
Absorbing boundary conditions & perfectly matched layering (PML)





Parallel computing (MPI partitions)



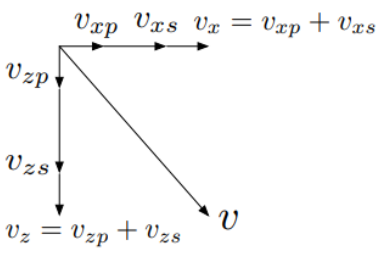


S3I highlights

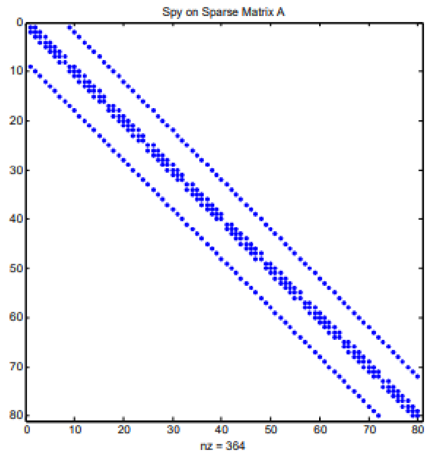
Extended seismic elastic wave modeling in time and frequency domains based on split P & S wave velocity and displacement fields for effective image migration and inversion

Elastic wave equation solution
and velocity field separation

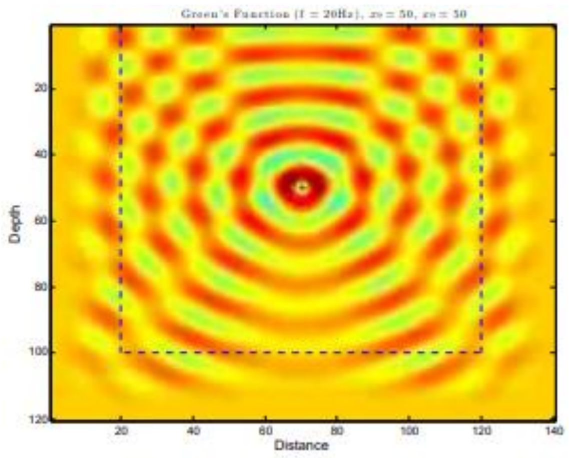
$$\frac{\partial v_{zp}}{\partial t} = \alpha^2 \frac{\partial A}{\partial z}, \quad \frac{\partial v_{xp}}{\partial t} = \alpha^2 \frac{\partial A}{\partial x}$$
$$\frac{\partial v_{zs}}{\partial t} = -\beta^2 \frac{\partial B}{\partial x}, \quad \frac{\partial v_{xs}}{\partial t} = \beta^2 \frac{\partial B}{\partial z}$$
$$A = \frac{\partial s_z}{\partial z} + \frac{\partial s_x}{\partial x}, \quad B = \frac{\partial s_x}{\partial z} - \frac{\partial s_z}{\partial x}$$



Sparse finite difference matrix
(freq. domain)



Green function solution
(20Hz example at selected PML)



S3I highlights

Accelerated Kirchhoff migration using 3D travel time imaging solutions to Eikonal equations for effective image migration and inversion

Kirchhoff migration and Eikonal equation

$$I(\xi) = \int_{\Omega_\xi} W(\xi, \boldsymbol{m}, \boldsymbol{h}) dt(t = t_d(\xi, \boldsymbol{m}, \boldsymbol{h}), \boldsymbol{m}, \boldsymbol{h}) d\boldsymbol{m} d\boldsymbol{h}$$

$$|\nabla T(\boldsymbol{x})| v(\boldsymbol{x}) = 1, \quad \boldsymbol{x} \in \mathbb{R}^2$$

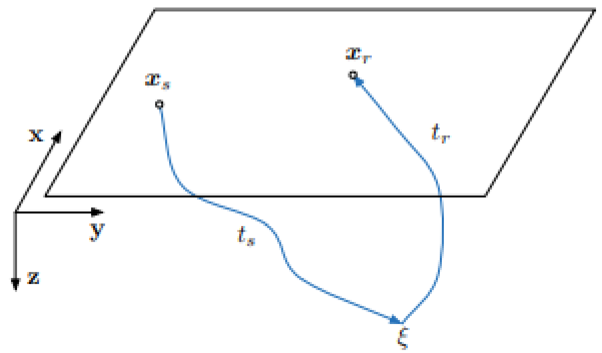
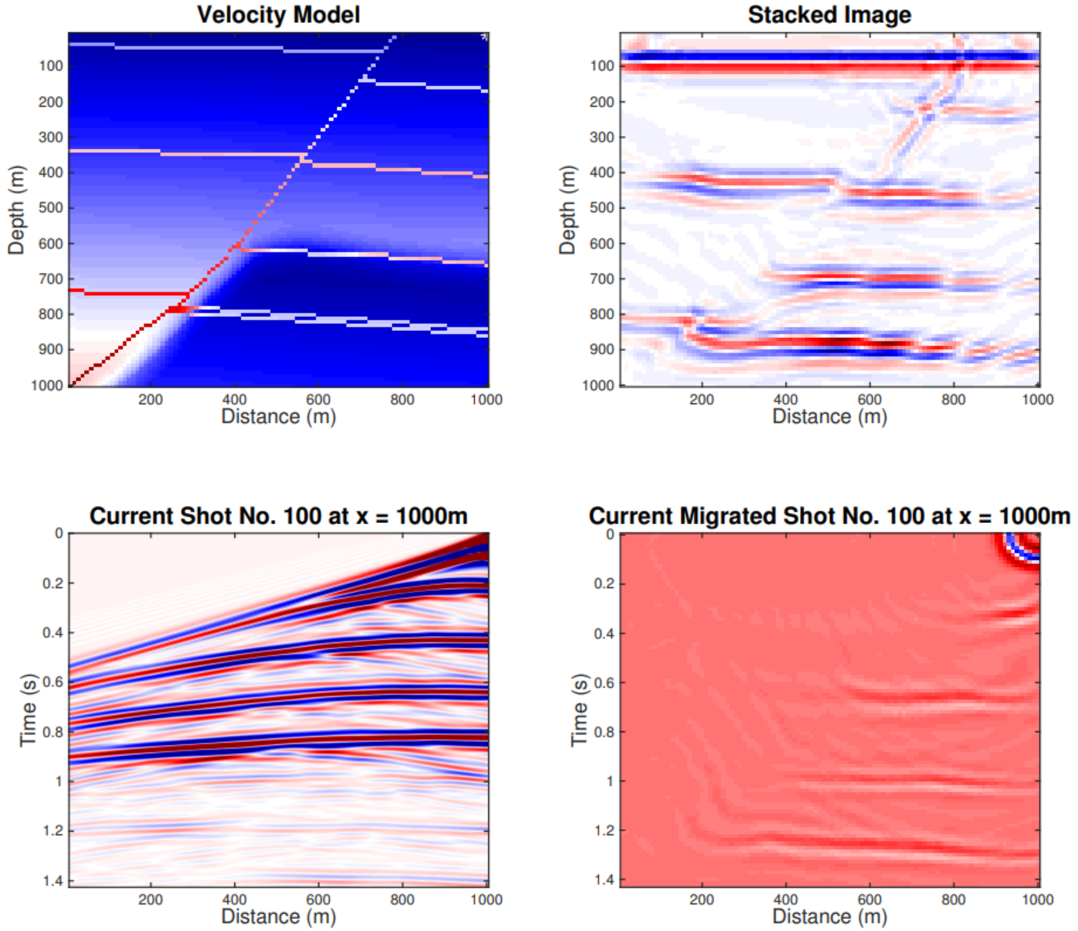


Figure 10: Reflector, and travel time



S3I highlights

Optimized Reverse-Time Migration (RTM) using forward and reverse wave propagation steps to maximize seismic imaging of complex geology with no dip limitation

Reverse time migration (RTM) workflow

Record forward wavefield through true velocity model $p_f(\boldsymbol{x}, t; \boldsymbol{x}_s)$

↓

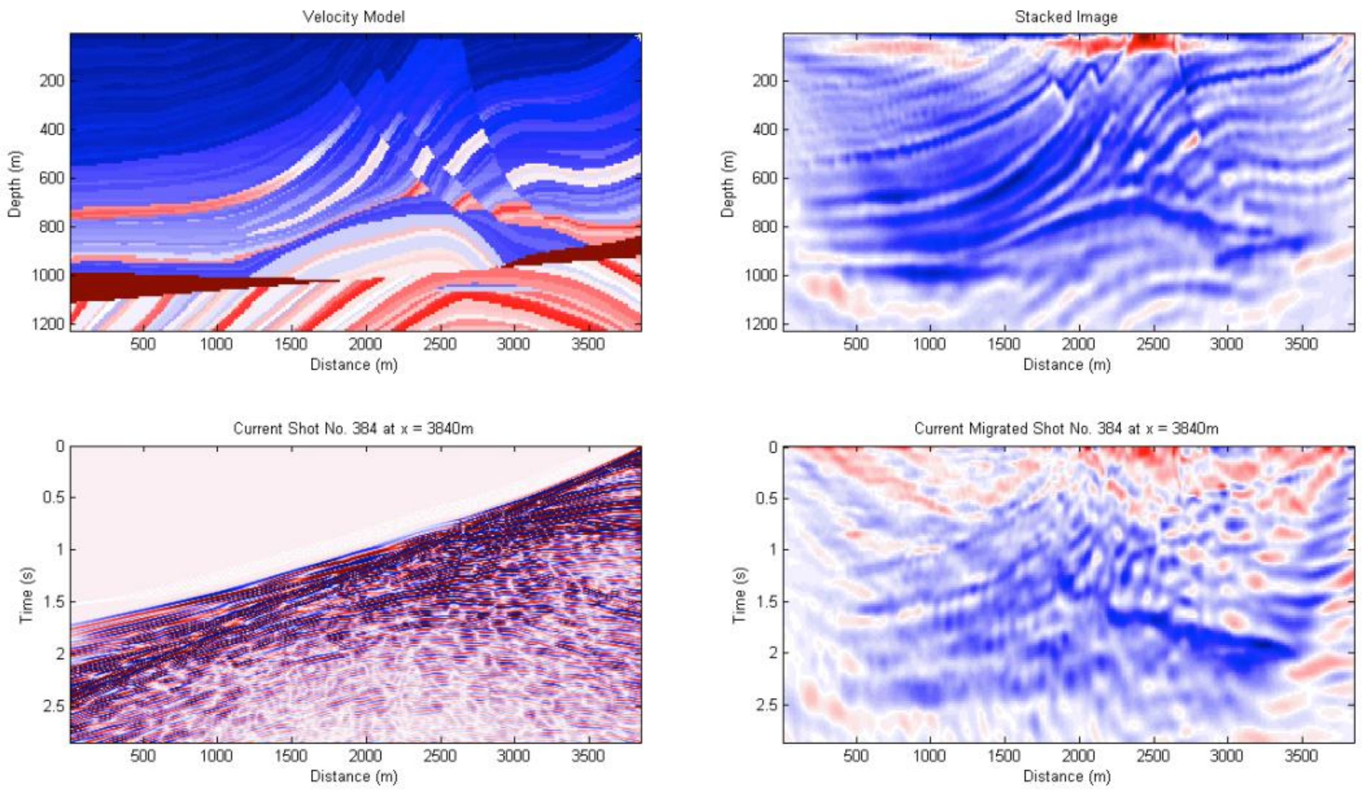
Solve reverse wavefield through incident model $p_r(\boldsymbol{x}, t)$

↓

Combine solution using an imaging condition (e.g., cross-correlation):

$$I(\boldsymbol{x}) = \sum_{\boldsymbol{x}_s} \frac{\int_0^T p_f(\boldsymbol{x}, t; \boldsymbol{x}_s) p_r(\boldsymbol{x}, t) \mathrm{d}t}{\int_0^T |p_f(\boldsymbol{x}, t; \boldsymbol{x}_s)|^2 \mathrm{d}t + \epsilon^2}$$

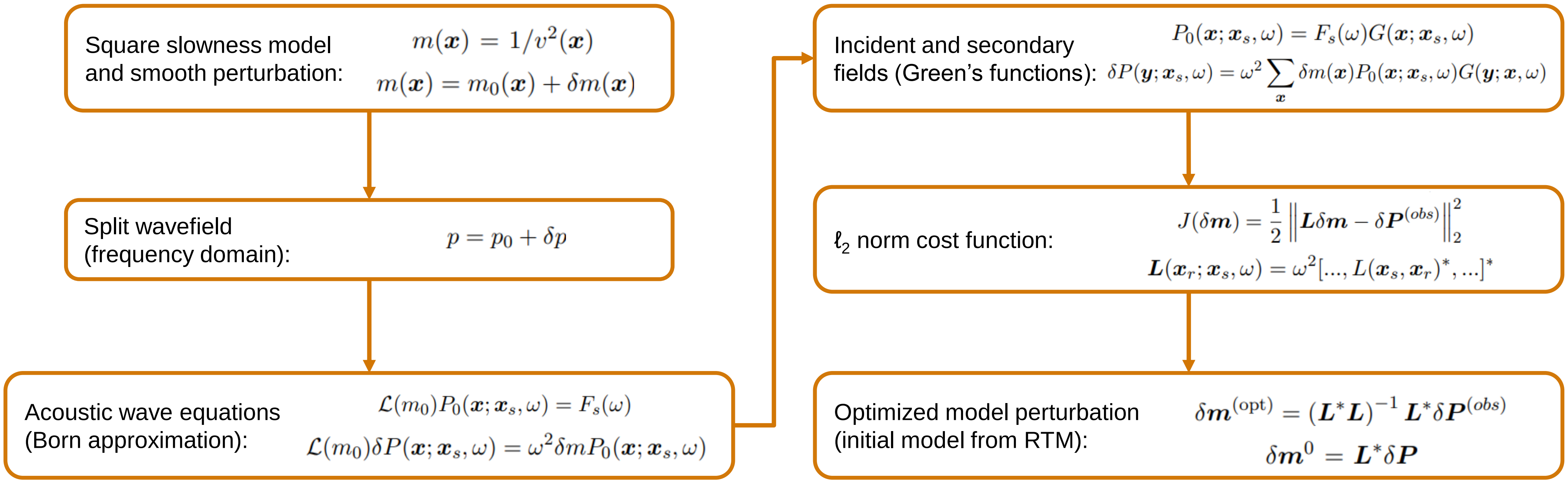
Reverse time migration (RTM) of Marmousi model (15Hz wavelet)



S3I highlights

Extended Least-Squared Migration (LSM) in both time and frequency domains using optimized Gauss-Newton solvers to accelerate migration workflow in broadband data

Least squares migration (LSM) workflow



S3I highlights

Broadband Full Waveform Inversion (FWI) using solutions to Green's functions in frequency domain based on projected Quasi-Newton optimization for fast conversion

Full waveform (FWI) workflow

Cost function for source and receive Green's functions:

$$J(\delta\boldsymbol{m}) = \frac{1}{2}\sum_{\omega,\boldsymbol{x}_s,\boldsymbol{x}_r}\left|\omega^2 F_s(\omega)\sum_{\boldsymbol{x}}\delta m(\boldsymbol{x})G_\omega(\boldsymbol{x};\boldsymbol{x}_s)G_\omega(\boldsymbol{x}_r;\boldsymbol{x})\right|$$

↓

Full-band scattering wavefield:

$$\delta\boldsymbol{P}_\omega^{(obs)} = \boldsymbol{P}_\omega^{(\mathrm{obs})} - \boldsymbol{G}_\omega(m)\boldsymbol{F}_\omega \text{ for all } \omega$$

↓

ℓ_2 norm optimization:

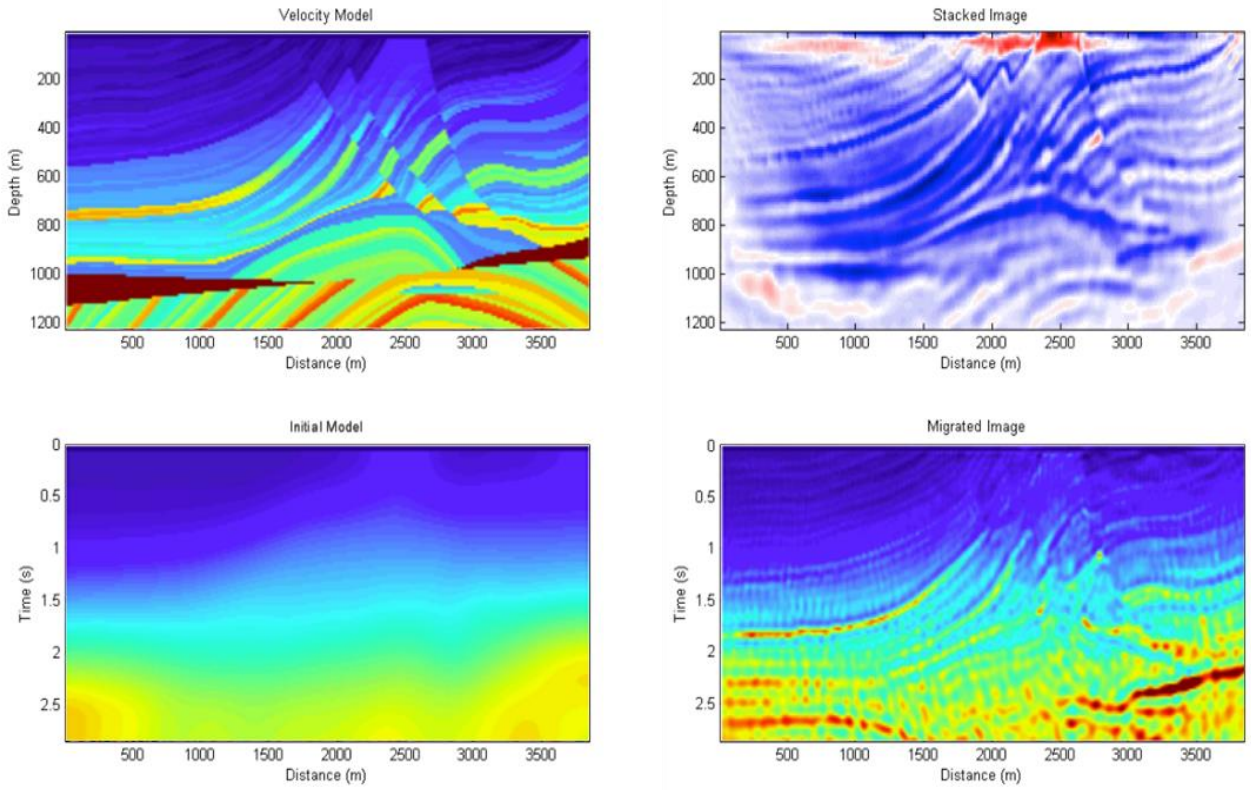
$$\delta\boldsymbol{m}^{(\mathrm{opt})} = \arg\min J(\delta\boldsymbol{m}) = \arg\min \tfrac{1}{2}\left\|\boldsymbol{L}(\boldsymbol{m})\delta\boldsymbol{m} - \delta\boldsymbol{P}^{(obs)}\right\|_2^2$$

↓

Model perturbation for convergence:

$$\boldsymbol{m} \leftarrow \boldsymbol{m} + \delta\boldsymbol{m} \qquad \delta\boldsymbol{m}^0 = \boldsymbol{L}^*\delta\boldsymbol{P}$$

Full Waveform Inversion (FWI) of Marmousi model (15Hz wavelet)



Accelerating seismic depth imaging using MATLAB GPU computing

S. Kozola | MathWorks (US)

Challenge

- Accelerate seismic migration workflows in S3I using parallel computing (GPU-based).

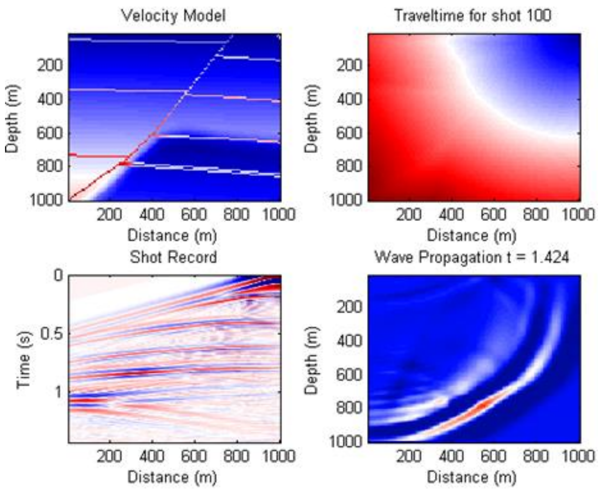
Solution

- Implemented a MATLAB workflow for seismic migration using both Kirchhoff and reverse time algorithms and a GPU extension based on a CUDA kernel to compare CPU & GPU solutions

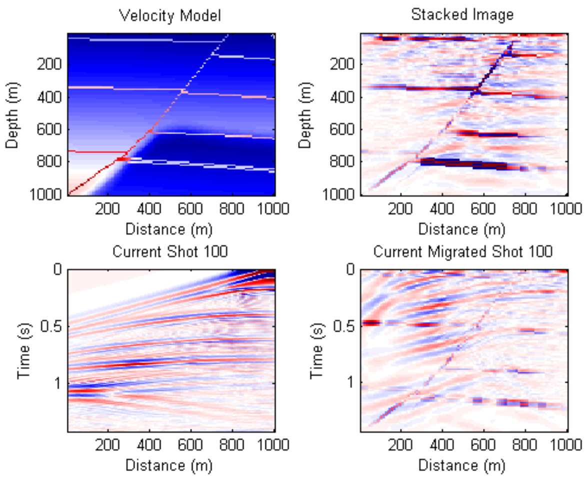
Benefit

- The custom CUDA kernel solution to the seismic wave equation using PDE finite differences was 1.6X faster than the vectorized CPU solution
- This early implementation of parallel computing in S3I helped to accelerate big data analysis.

Seismic pre-stack wave model



Seismic RTM migration stack



MATLAB Seismic migration & GPU computing workflow

3D seismic dataset (40Tb)

- Seismic waveform modeling
- Kirchhoff and reverse time migrations
- CPU (vectorized) vs. GPU (CUDA) solution

RTM workflow runs 1.6X-1.8X faster on GPU

Source: S. Kozola (2011). Large data in MATLAB: A seismic data processing case study. MATLAB Central File Exchange. https://www.mathworks.com/matlabcentral/fileexchange/30585-large-data-in-matlab-a-seismic-data-processing-case-study

Accelerating seismic facies classification using PI-RNNs

Mishra (2018) | MathWorks (US)

Challenge

- Accelerate seismic facies classification using AI and parallel computing (CPU & GPU-based).

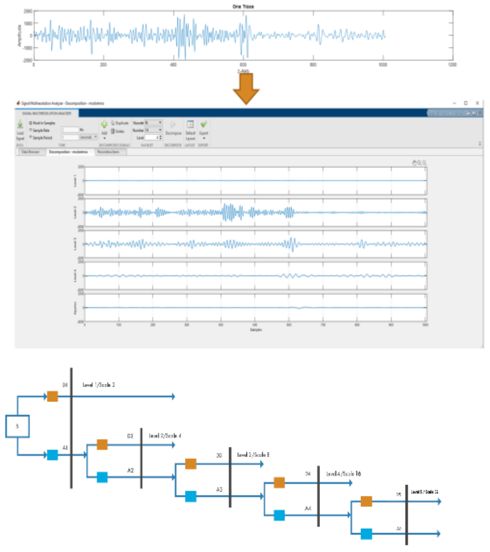
Solution

- Implemented a MATLAB workflow for seismic signal processing based on a physics-informed recurrent neural networks (PI-RNN) using LSTM and discrete wavelet decomposition to enhance and accelerate seismic facies classification using multi-core and GPU CUDA kernel solutions.

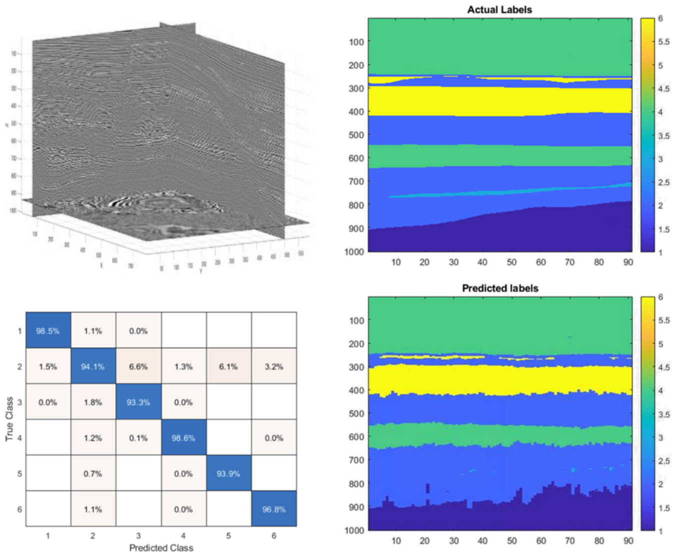
Benefit

- The PI-RNN workflow predicted seismic facies with 76% correlation, ~2X higher than CNN
- The CUDA kernel solution was ~70X faster (~3min) than vectorized CPU solution (~3 hrs).

Discrete wavelet transform



Seismic facies labels and PI-RNN prediction

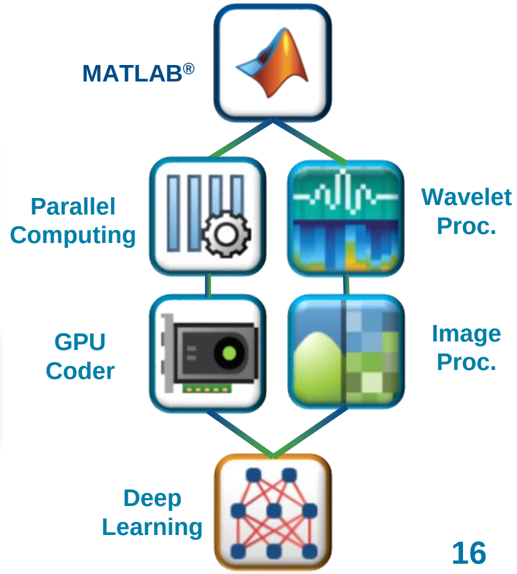


MATLAB Seismic facies classification & GPU computing workflow

3D seismic dataset (200Gb)

- Signal-based RNN model using GRUs
- Discrete Wavelet Transform (DWT)
- CPU (vectorized) vs. GPU (CUDA) solution

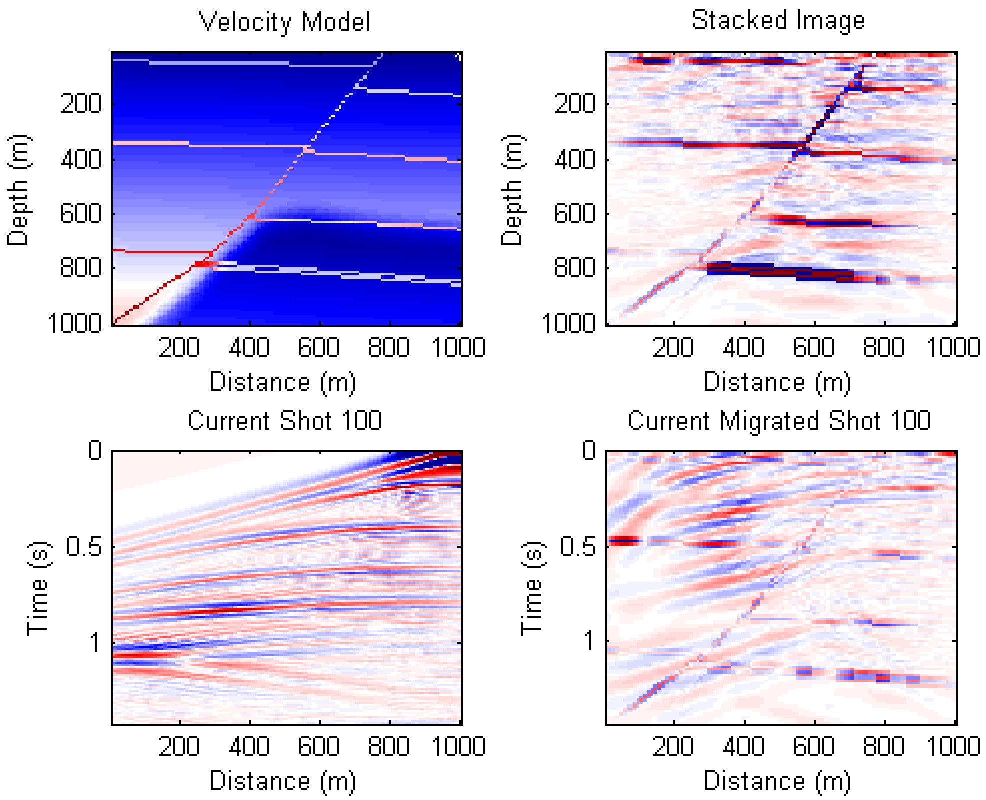
Facies correlated 76% with 93% trained data
RNN correlation ~2X higher than CNN (40%)
GPU time: 2.5 min vs. CPU time: 3 hours



Source: Mishra, A. (2018). Facies Classification with Wavelets and Deep Learning. MATLAB Energy Conference 2021. MathWorks.

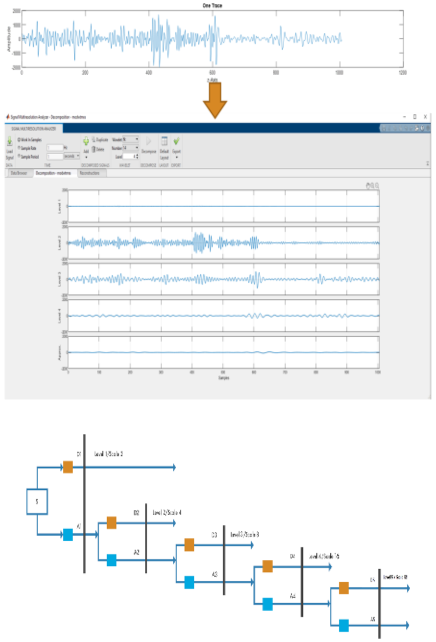
MathWorks® Seismic Processing and Imaging Resources

[Large Data in MATLAB: A Seismic Data Processing Case Study | File Exchange - MATLAB Central]

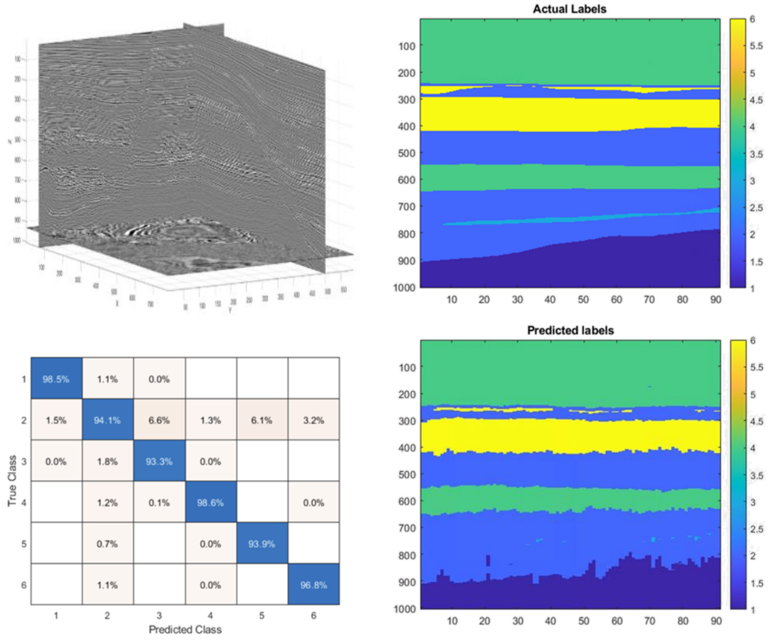


[Seismic Facies Classification with Physics-Informed Neural Networks (mathworks.com)]

Discrete wavelet transform



Seismic facies labels and PI-RNN prediction



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