**Damped Sinusoidal Motion or Damped Vibrations**

**Damped Sinusoidal Motion**

Many physical responses can be modeled by damped sinusoidal motion. For example, the shock absorbers in a car allow the passengers to experience damped vibrations when hitting a pothole or a speed bump. Consider a mass connected to a spring. If there were no loss of energy, compressing or stretching the spring would result in sustained oscillations (simple harmonic motion). However, in reality, there is a loss of energy in the spring and the amplitude of the oscillations will decrease over time.

Depending on the characteristics of the spring and the mass, the displacement of the mass will look like one of the two graphs shown below. The mass could oscillate around the equilibrium point with the amplitude of the oscillations becoming smaller and smaller (***underdamped response***) or the mass could simply return directly to the equilibrium point (***over-damped response***).



If the response is ***underdamped***, the displacement of the mass can be modeled mathematically as follows:

**d:** displacement of the mass in meters (m)

**d0:** initial displacement of the mass in meters (m)

**ω:** the frequency of oscillation (rad/s)

**M:** mass (kg)

**K:** spring constant (N/m)

**B:** damping coefficient (N**.**s/m)

The spring constant is a measure of resistance to displacement. The force that it takes to compress or stretch the spring is directly proportional to the displacement. In other words, it takes twice as much force to compress or stretch the spring by 10 cm than it would to compress of stretch the spring by 5 cm.

The damping coefficient models the energy loss in the system. The damping is proportional to the velocity of the mass. The faster the mass moves, the higher the damping force opposing that motion will be.

**Note: If , the response will be over-damped (the mass simply returns directly back to equilibrium if damping force is large enough).**

Write a program that will allow you to explore the effects of the spring constant, K, the damping coefficient, B, and the mass, M, on the motion of the mass.

1. Write four input statements to prompt the user for the spring constant, K, the mass, M, the damping coefficient, B, and the initial displacement, dint. Make sure you write good prompts for the user which include units. For example: input(‘Enter K: ‘) is not very informative to the user. input(‘Enter the spring constant, K, in N/m: ‘)is a much better prompt.
2. Write a conditional statement using the ***if …. else … end*** construction. ***If*** , your program should inform the user that the response is over-damped. Otherwise (***else***), your program should inform the user that the response is underdamped. Use an ***fprintf*** statement to do this. Display all values using two places behind the decimal point. For example, if the response turns out to be over-damped, your program should output the following statement:

For K = *display\_value\_entered\_by\_user*, B = *display\_value\_entered\_by\_user,* and M = *display\_value\_entered\_by\_user*, the response is over-damped.

1. Now test your program. Suppose K = 200 N/m, M = 0.5 kg, and d0 = 1 m.

Calculate the range of B that will result in an over-damped response? \_\_\_\_\_\_\_\_\_ (Nm/s)

Calculate the range of B will result in an under-damped response? \_\_\_\_\_\_\_\_\_\_\_\_\_ (Nm/s)

Run your program for various values of B and see if it produces the correct result.

1. In the underdamped section of your program (***else***), add a line to compute the frequency of the oscillations, ω, in radians per second.
2. Add another line to compute the frequency of the oscillations in Hz. (Recall: ω = 2πf).
3. Add an ***fprintf*** statement that tells the user what the frequency of oscillation is in both radians per second and in Hz. Display the frequencies using ***2 places*** behind the decimal point.
4. Now test your program again using the values K = 200 N/m, M = 0.5 kg, B = 4, and do = 1m. The frequency of oscillation should be 19.6 rad/s or 3.12 Hz.
5. The amplitude of the oscillations decay exponentially as

Find an equation for the time, T\_END, at which this amplitude will be: . Your

equation will depend on B and M.

T\_END = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (s)

Note: so when t = T\_END, the displacement of the mass will be less than 1% of the initial displacement and the oscillations will be pretty much damped out.

1. Add a line to your program (again in the ***else*** or underdamped section), to calculate the time, T\_END at which the oscillations are effectively damped out. Add an ***fprintf*** statement to tell the user how long it takes for the oscillations to essentially stop. Display this value using ***four places*** behind the decimal point.
2. Test your program again using the values K = 200 N/m, M = 0.5 kg, B = 4 and d0 = 1m. What should T\_END be? Does your program calculate and display this value correctly?
3. In the underdamped section of your code, add some lines of code to plot the underdamped response (time on the x-axis and displacement on the y-axis) from t = 0 to t = T\_END.

* Choose the increment for t based on T\_END
* The plot should be a solid black line – format this within your code, not with plot tools.
* Label the x and y axis within your program – not with plot tools. Include units.
* Add a title as part of your program code – not with plot tools.

1. Test your program again using the values K = 200 N/m, M = 0.5 kg, B = 4, and d0 = 1 m.

Paste the plot in the space below. Also paste the output from your program that appears in the command window.

**MATLAB PLOT (K = 200 N/m; M = 0.5 kg; B = 4):**

**OUTPUT FROM fprintf STATEMENTS:**

1. Use your program to complete the following table. Use d0 = 1 m for all cases. You only need to include copies of the plots for the cases highlighted in yellow in the table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **K (N/m)** | **M (kg)** | **B (Ns/m)** | **Type of Response** | **ω (rad/s)** | **f (Hz)** | **Approximate**  **end time (sec)**  **of oscillations** |
| **200** | **0.5** | **4** |  |  |  |  |
| **400** | **0.5** | **4** |  |  |  |  |
| **800** | **0.5** | **4** |  |  |  |  |
| **800** | **0.5** | **8** |  |  |  |  |
| **800** | **0.5** | **16** |  |  |  |  |
| **800** | **0.5** | **32** |  |  |  |  |
| **800** | **0.5** | **40** |  |  |  |  |
| **400** | **1** | **4** |  |  |  |  |
| **400** | **2** | **4** |  |  |  |  |
| **400** | **3** | **4** |  |  |  |  |

**MATLAB PLOT: K = 800 N/m; M = 0.5 kg; B = 4 Ns/m**

**MATLAB PLOT: K = 800 N/m; M = 0.5 kg; B = 32 Ns/m**

**MATLAB PLOT: K = 400 N/m; M = 3 kg; B = 4 Ns/m**

1. Describe how increasing the spring constant affects the response.
2. Describe how increasing the damping coefficient affects the response.
3. Describe how increasing the mass affects the response.
4. If you have not done so already, add comments to your script file to explain what your program does.

**Comment:** We have investigated harmonic motion (sustained oscillations) and damped sinusoidal motion. There is another type of response, generally undesirable, where the amplitude of the oscillations increases with time. Terms for this type of response include self-excited oscillations or aeroelastic flutter and can lead to destructive vibrations in a structure or an aircraft. In 1940, the Tacoma Narrows Bridge literally shook itself apart due to a self-excited oscillation induced by wind. You can watch the video here: <http://www.youtube.com/watch?v=xox9BVSu7Ok> Aeroelastic flutter has also been the cause of several aircraft crashes where the wing of the aircraft began to vibrate so violently that it broke off the airplane. Avoidance of aeroelastic flutter is an extremely important consideration in the design of aircraft, bridges, and other structures.