# Embedded Control and Mechatronics

**Spring 2012** | **ECE-456**

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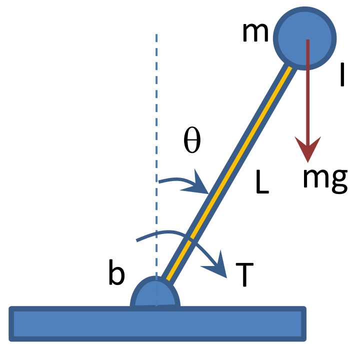
(**Solve the problems, simulate and verify with Matlab/Simulink, show simulation results and plots**)

#1) Consider an upright pendulum, with m=0.2 kg, L=0.5 m, and friction coefficient b=0.2 Ns/m, as shown in the figure.




a) Select a state vector $x$ and write the **state equation** of the system, where $y = \theta$ is the output and $u = T$ is the input torque.
b) Develop a **Simulink model** of the system.
c) Find the **operating/equilibrium point** of the system for the three cases of $y = -\pi/4$, $y = 0$, and $y = \pi/4$.
d) Write the **linearized state-space model** of the system around these equilibrium-points, and find the **transfer function** of these linearized models.
e) For the linearized model at the equilibrium point $y = 0$, design a **lead-lag compensator** so that, for a unit-step reference command, the cross-over frequency, phase-margin, and position-error constant of the closed-loop system have the desired values of $\omega_c = 10$, $PM = 70°$, and $K_p = 20$.
f) **Draw and compare the simulation plots** of the state, input, and output variables of the closed-loop system, when the above compensator/controller is applied to both the linearized model and the actual nonlinear model. **Modify your controller**, if necessary, to achieve the desired closed-loop response for the actual nonlinear system.

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#2) Consider an LTI system "sys", given as $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$, where

$$A = \begin{bmatrix} 2.25 & -5 & -1.25 & -0.5 \\ 2.25 & -4.25 & -1.25 & -0.25 \\ 0.25 & -0.5 & -1.25 & -1 \\ 1.25 & -1.75 & -0.25 & -0.75 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 4 \\ 2 \\ 2 \end{bmatrix}, C = [0 \quad 2 \quad 0 \quad 2], \text{ and } D = [0].$$

a) Find the **eigenvalues** of the system "sys" (i.e., poles of its transfer function matrix) and determine the **stability** of the system.
b) Find the second-order **balanced realization** of the system, "sysr2".
c) Using "System Identification" toolbox and applying "PRBS" inputs, **estimate** a second-order model and a fourth-order model of the system, "sysID2" and "sysID4".
d) Draw and compare the **Bode-plots and step-responses** of these four system models.
e) Design a **lead-lag compensator** for the approximate second-order system model "sysr2", so that, for a unit-step reference command, the cross-over frequency, phase-margin, and position-error constant of the closed-loop system have the desired values of $\omega_c = 5$, $PM = 65°$, and $K_p = 10$.
f) **Draw and compare the simulation plots** of the state, input, and output variables of the closed-loop system, when the above compensator/controller is applied to all four models of the system. **Modify your controller**, if necessary, to achieve the desired closed-loop response for the original system "sys".

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