Model-Based Controller Design

Linear control design techniques

- Three major steps for control design
  - Modeling, analysis, and design (MAD) process
- The control design is based on system's model (**model-based**)
- The closed-loop system must meet certain design specifications

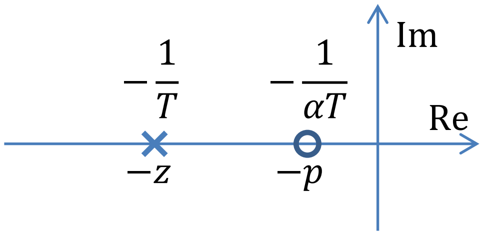
- Model-based design techniques
  - Cascade lead-lag compensator design
  - Optimal linear quadratic regulator (LQR)
  - Pole placement design
  - Decoupling control of multivariable systems

- SISOTool control design environment

Lead-Lag Compensator Design

Phase-lead compensator

$$G_c(s) = \frac{U(s)}{E(s)} = K\frac{(s+z)}{(s+p)} \quad ; \quad z < p \quad (lead)$$



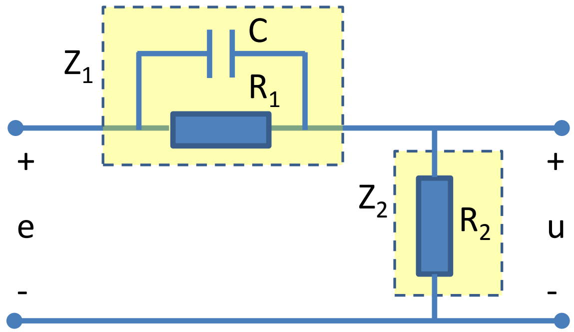
- Pole on the left side of zero

- Equivalent RC circuit, K=1, ($Z_1 = \frac{R_1}{1+R_1Cs}$, $Z_2 = R_2$)

$$G_c(s) = \frac{U(s)}{E(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{1}{\alpha}\frac{(1+\alpha Ts)}{(1+Ts)}$$

$$T = \frac{R_1R_2}{R_1+R_2}C\,, \quad \alpha = \frac{R_1+R_2}{R_2} > 1$$

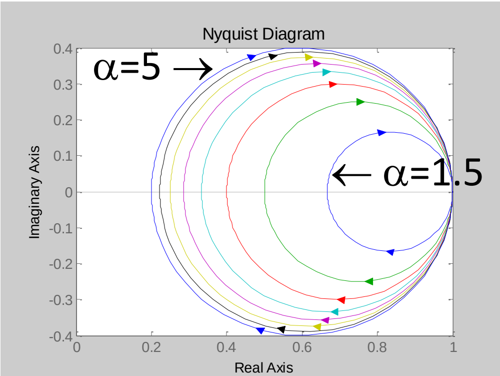
Or: $\quad p = \frac{1}{T}, \quad z = \frac{1}{\alpha T}$

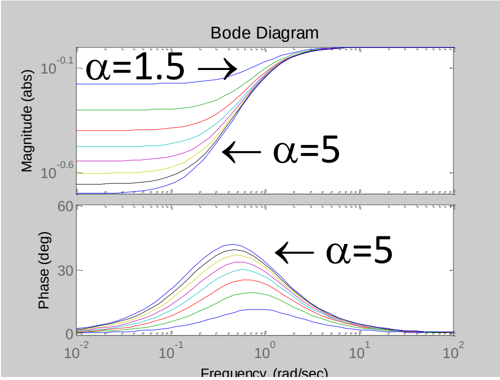


- Bode and Nyquist plots of lead compensator

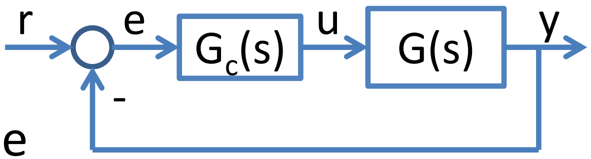
  T=1

  α=1.5:0.5:5





Lead-Lag Compensator Design



Example (lead compensator):

For $G(s) = \dfrac{100}{s(0.04s+1)}$, increase the closed-loop phase

Phase margin: PM=28.0243° at ω_{cp}=46.9701 rad/sec

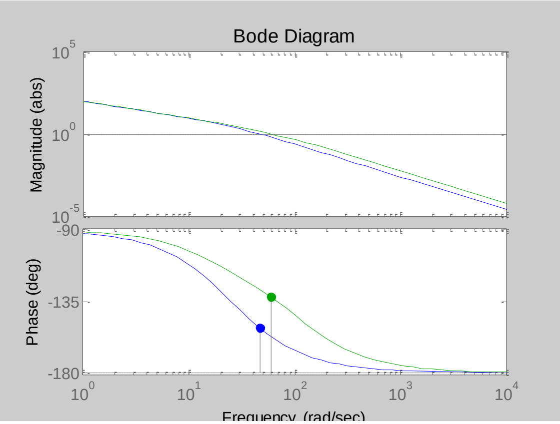
- Consider the lead compensator $G_c(s) = \dfrac{(1+0.0262s)}{(1+0.0106s)}$
- Draw and compare the Bode plots and step responses

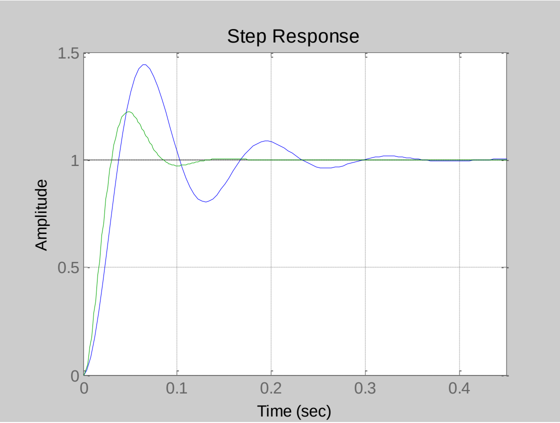
Matlab code:

```
G=tf(100,[0.04,1,0]);
[Gm,Pm,wcg,wcp]=margin(G),
Gc1=tf([0.0262,1],[0.0106,1]); GGc1=G*Gc1;
[Gm1,Pm1,wcg1,wcp1]=margin(GGc1),
figure(1), bode(G,GGc1); grid off;
Gcl=feedback(G,1,-1);
Gcl1=feedback(GGc1,1,-1);
figure(2), step(Gcl,Gcl1);
```

Phase margin with lead compensation:

PM=47.6° at ω_{cp}=60.3 rad/sec



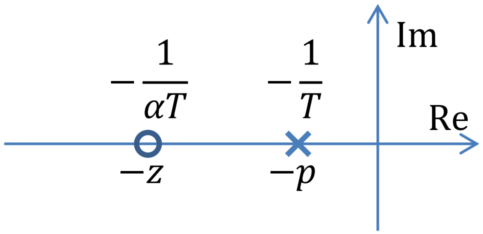


Lead-Lag Compensator Design

Phase-lag compensator

$$G_c(s) = \frac{U(s)}{E(s)} = K\frac{(s+z)}{(s+p)} \quad ; \quad z > p \quad (lag)$$

- zero on the left side of pole

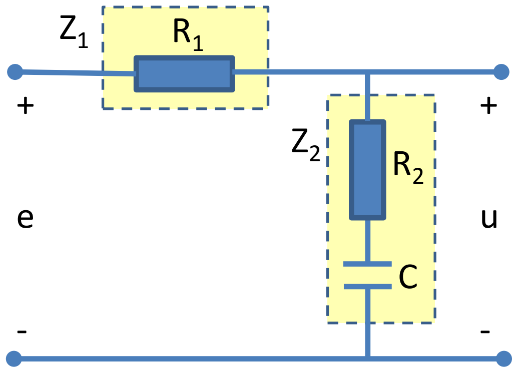


- Equivalent RC circuit, K=1, $(Z_1 = R_1,\ \ Z_2 = R_2 + \frac{1}{Cs})$

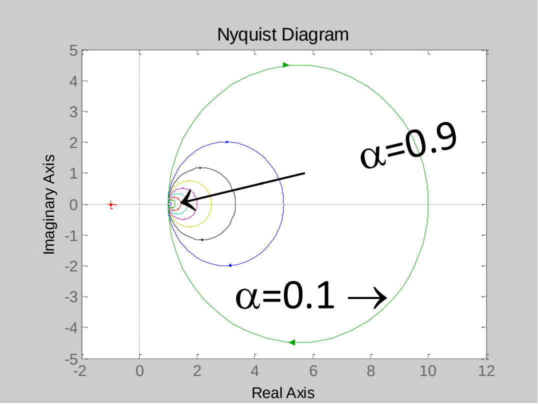
$$G_c(s) = \frac{U(s)}{E(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{1+\alpha Ts}{1+Ts}$$

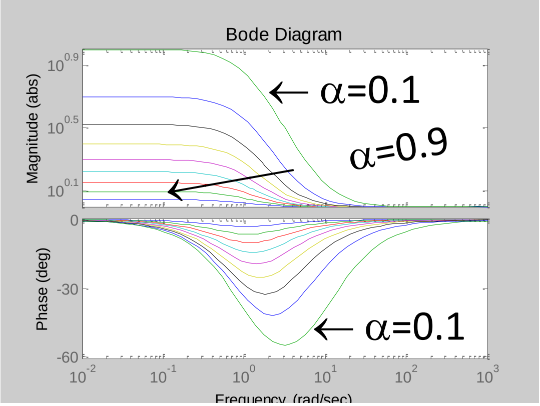
$$T = (R_1 + R_2)C\ , \quad \alpha = \frac{R_2}{R_1+R_2} < 1$$

Or: $\quad p = \frac{1}{T}, \ \ z = \frac{1}{\alpha T}, \ \ K = \alpha$



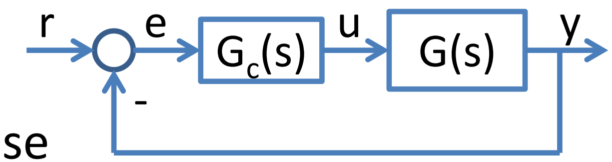
- Bode and Nyquist plots of lag compensator
  T=1, K=1
  α=0.9:-0.1:0.1





Lead-Lag Compensator Design

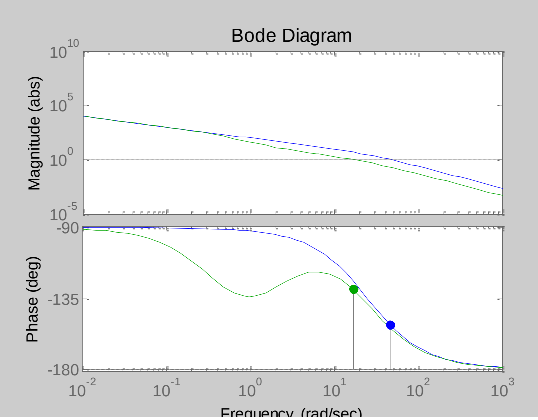
Example (lag compensator):



For $G(s) = \dfrac{100}{s(0.04s+1)}$, increase the closed-loop phase

Phase margin: PM=28.0243° at ω_{cp}=46.9701 rad/sec

- Consider the lag compensator $G_c(s) = \dfrac{(1+0.5s)}{(1+2.5s)}$
- Draw and compare the Bode plots and step responses

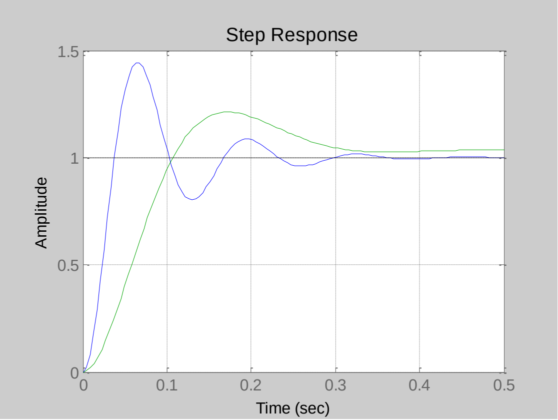


Matlab code:

```
G=tf(100,[0.04,1,0]);
[Gm,Pm,wcg,wcp]=margin(G),
Gc2=tf([0.5,1],[2.5,1]); GGc2=G*Gc2;
[Gm2,Pm2,wcg2,wcp2]=margin(GGc2),
figure(1), bode(G,GGc2); grid off;
Gcl=feedback(G,1,-1);
Gcl2=feedback(GGc2,1,-1);
figure(2), step(Gcl,Gcl2);
Gc1=tf([0.0262,1],[0.0106,1]);
GGc1=G*Gc1;
Gcl1=feedback(GGc1,1,-1);
figure(3), step(Gcl,Gcl2,Gcl1);
```

Phase margin with lead compensation:

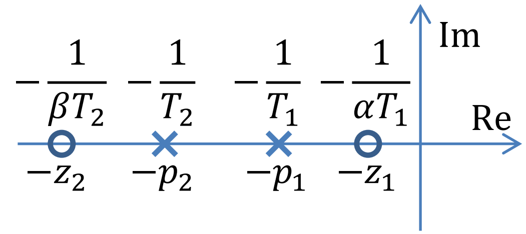
PM=47.6° at ω_{cp}=60.3 rad/sec



Lead-Lag Compensator Design

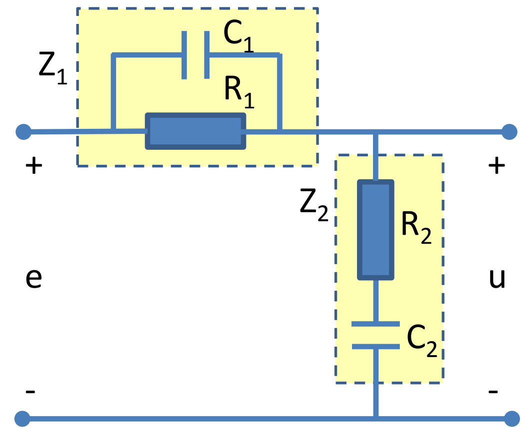
Phase lead-lag compensator

$$G_c(s) = \frac{U(s)}{E(s)} = K\frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}\ ; \quad \begin{array}{ll} z_1 < p_1 & (lead) \\ z_2 > p_2 & (lag) \end{array}$$



- For lead-lag, (z_1, p_1) is on the left of (z_2, p_2)
- For lag-lead, (z_1, p_1) is on the right of (z_2, p_2)

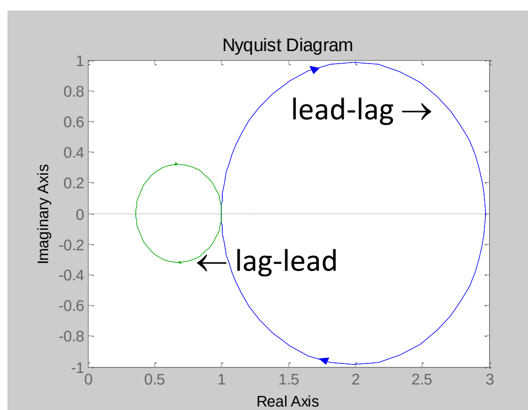
- Equivalent RC circuit, $\left(Z_1 = \frac{R_1}{1+R_1C_1s},\ \ Z_2 = R_2 + \frac{1}{C_2s}\right)$

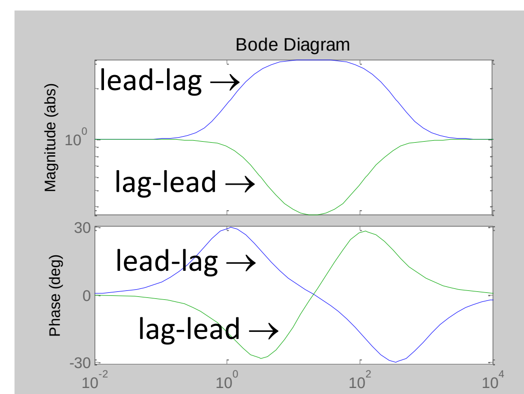


$$G_c(s) = \frac{U(s)}{E(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{(1+\alpha T_1 s)(1+\beta T_2 s)}{(1+T_1 s)(1+T_2 s)}$$

$$\alpha T_1 = R_1C_1,\ \ \beta T_2 = R_2C_2,\ \ \alpha\beta = 1 \quad \alpha > 1,\ \ \beta < 1$$
$$T_1 + T_2 = R_1C_1 + R_2C_2 + R_1C_2,\ \ T_1T_2 = R_1C_1R_2C_2$$

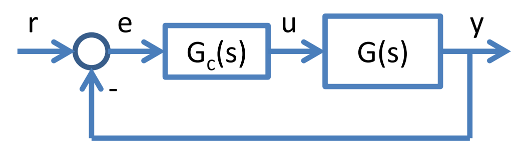
- Bode and Nyquist plots of **lead-lag** (T1=0.5, T2=0.005) and **lag-lead** (T2=0.5, T1=0.005) compensators with (α=3, β=1/3)





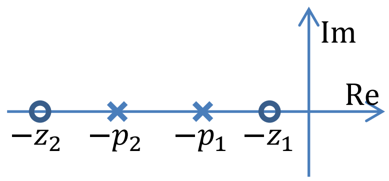
Lead-Lag Compensator Design

Lead-lag design by phase-margin assignment



Given Plant model: $G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{s^\nu (a_0 s^{n-\nu} + a_1 s^{n-\nu-1} + \cdots + a_{n-\nu+1}s + a_{n-\nu})}$

Design compensator: $G_c(s) = K\frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$; $z_1 \le p_1 \quad (lead)$
$z_2 \ge p_2 \quad (lag)$



Design specifications:

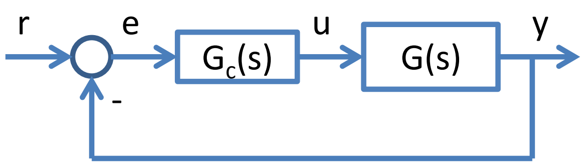
- Desired cross-over frequency: ω_{cd}
- Desired phase-margin of the compensated system at ω_{cd}: PM_d
- Desired error-constant: greater than or equal to K_{ed}

Denote:

- Error constant: $K_e = \lim_{s \to 0} s^\nu G_o(s)$, where ν is the number of poles of G(s) at the origin, $G_o(s) = G_c(s)G(s)$
- Phase-angle and magnitude of the plant model at ω_{cd}: $\phi_g(\omega_{cd})$, $A(\omega_{cd})$
- Phase-angle of the compensator at ω_{cd}: $\phi_c(\omega_{cd}) = PM_d - 180 - \phi_g(\omega_{cd})$

Lead-Lag Compensator Design

Lead-lag design procedure



$$\phi_c(\omega_{cd}) = PM_d - 180 - \phi_g(\omega_{cd})$$

Case 1: If $\phi_c(\omega_{cd}) > 0$, a lead compensator is needed as

a) Let $\alpha = \frac{z_1}{p_1} = \frac{1-sin\phi_c}{1+sin\phi_c} > 1$, then $z_1 = \sqrt{\alpha}\,\omega_{cd}$, $p_1 = \frac{z_1}{\alpha} = \frac{\omega_{cd}}{\sqrt{\alpha}}$, $K = \frac{\sqrt{{\omega_{cd}}^2+{p_1}^2}}{\sqrt{{\omega_{cd}}^2+{z_1}^2}\;A(\omega_{cd})}$

- The error-constant of compensated system is: $K_e = \lim_{s\to 0} s^{\nu} G_c(s) G(s) = \frac{b_m}{a_{n-\nu}} \frac{K\,z_1}{p_1}$
  - If $K_e \geq K_{ed}$, lead compensator is sufficient
  - If $K_e < K_{ed}$, need lead-lad compensator

Case 2: If $\phi_c(\omega_{cd}) < 0$, a lag compensator is needed as

a) Let $K = \frac{1}{A(\omega_{cd})}$, $z_2 = \frac{\omega_{cd}}{\beta}$, $p_2 = \frac{K_e}{K_{ed}} z_2$, where $\beta \in [4,10]$, $K_e = \lim_{s\to 0} s^{\nu} G(s) = \frac{b_m}{a_{n-\nu}} K$

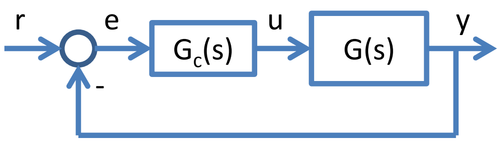
Case 3: After lead compensation design, if error-constant specification is not satisfied ($K_e = \lim_{s\to 0} s^{\nu} G_c(s) G(s) < K_{ed}$), a lead-lag compensator is needed, consisting of the above phase-lead design and a phase-lag as

a) Select z_1, p_1, K as in case 1. Then, $z_2 = \frac{\omega_{cd}}{\beta}$, $p_2 = \frac{K_e}{K_{ed}} z_2$, where $\beta \in [4,10]$,

$$K_e = \lim_{s\to 0} s^{\nu} G_o(s) = \frac{b_m}{a_{n-\nu}} \frac{K\,z_1}{p_1}$$

Lead-Lag Compensator Design

Lead-lag design with Matlab



Matlab function: leadlag(),

Syntax: Gc=leadlag(G,wc,PMd,Ke,key)

- G is he plant model
- wc is the desired crossover frequency
- PMd is the desired phase-margin
- Ke is the static error constant
- key=1,2,3 for cases 1,2,3
- Gc is the TF of the compensator

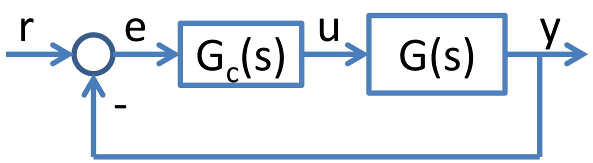
Matlab code:

```
% Lead-lag compensator design
function Gc=leadlag(G,wc,Gam_c,Kv,key)
G=tf(G); [Gai,Pha]=bode(G,wc); Phi=Gam_c-Pha-180; N=1;
if Phi<0, Ph0=mod(-Phi,70); N0=-(Phi+Ph0)./70; N=N0+1; end,
Phi_c=Phi*pi/180/N; den=G.den{1}; a=den(length(den):-1:1);
ii=find(abs(a)<=0); num=G.num{1}; G_n=num(end);
if length(ii)>0, a=a(ii(1)+1); else, a=a(1); end; a=abs(a);
alpha=sqrt((1-sin(Phi_c))/(1+sin(Phi_c))); Zc=alpha*wc;
Pc=wc/alpha;
Kc=sqrt(((wc*wc+Pc*Pc)/(wc*wc+Zc*Zc))^N)/Gai;
K1=G_n*Kc*alpha^2/a;
if nargin==4, key=1;
   if Phi_c<0, key=2; else if K1<Kv, key=3; end, end, end
switch key
   case 1, Gc=(tf([1,Zc]*Kc,[1,Pc]))^N;
   case 2, Kc=1/Gai; K1=G_n*Kc/a;
Gc=tf([1,0.1*wc],[1,0.1*K1*wc/Kv]);
   case 3, Zc2=wc*0.1; Pc2=K1*Zc2/Kv; Gcn=Kc*conv(poly(-
Zc*ones(N,1)),[1,Zc2]);
      Gcd=conv(poly(-Pc*ones(N,1)),[1,Pc2]); Gc=tf(Gcn,Gcd);
end
```

Lead-Lag Compensator Design

Example:

For $G(s) = \dfrac{100}{s(0.04s+1)}$, design lead-lag compensator

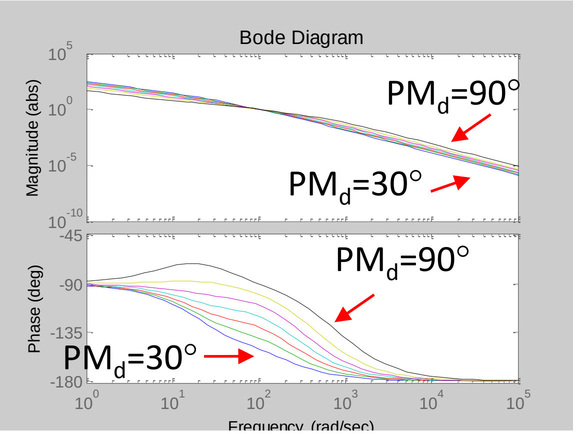


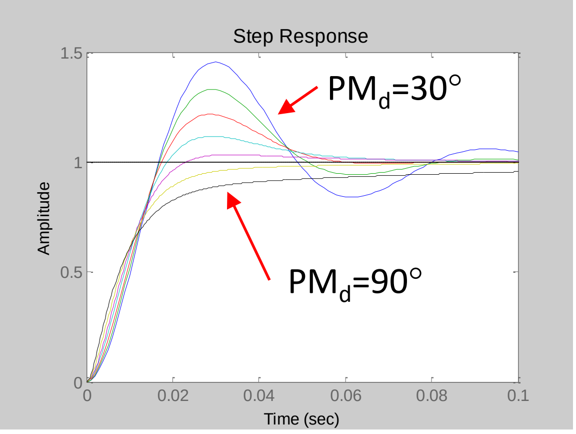
- Desired phase margin: PM_d=[30,40,…,90]°
- Desired crossover frequency: ω_{cd}=100 rad/sec
- Desired error constant: $K_{ed} \geq 10$

- Draw and compare the Bode plots and step responses

Matlab code:

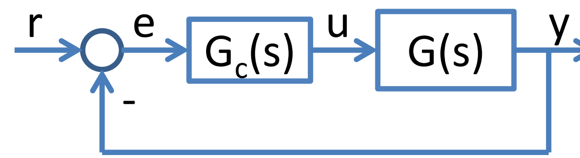
```
G=tf(100,[0.04,1,0]);  wcd=100;  Ked=10;
for pmd=[30,40,50,60,70,80,90],
    Gc=leadlag(G,wcd,pmd,Ked);
    Go=Gc*G; Gcl=feedback(Go,1);
    figure(1), bode(Go); hold on; grid off;
    figure(2), step(Gcl,0.1); hold on; grid off;
end
```





Lead-Lag Compensator Design

Example:

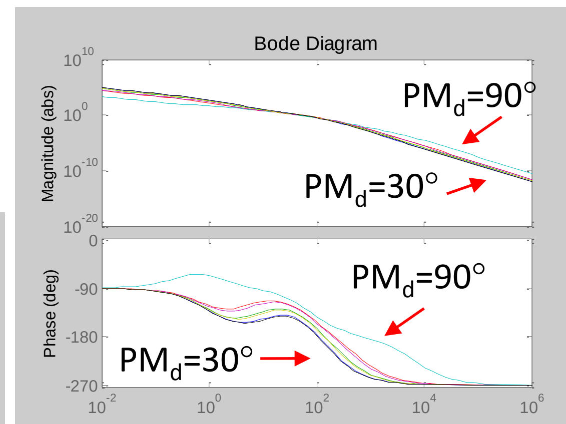


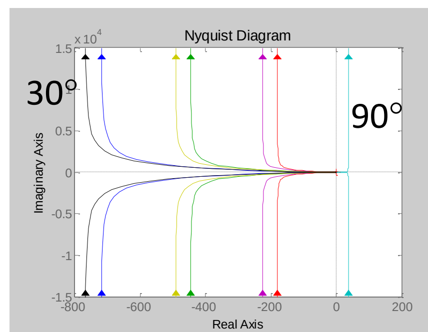
Plant model: $G(s) = \frac{100}{s(s+1)(0.0125s+1)}$

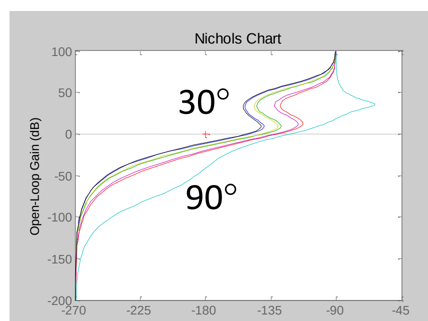
- Design lead-lag compensator
  - Desired phase margin: PM_d=[30,40,50,60,70,80,90]°
  - Desired crossover frequency: ω_{cd}=50 rad/sec
  - Desired error constant: $K_{ed} \geq 10$
- Draw and compare the Bode plots and step responses

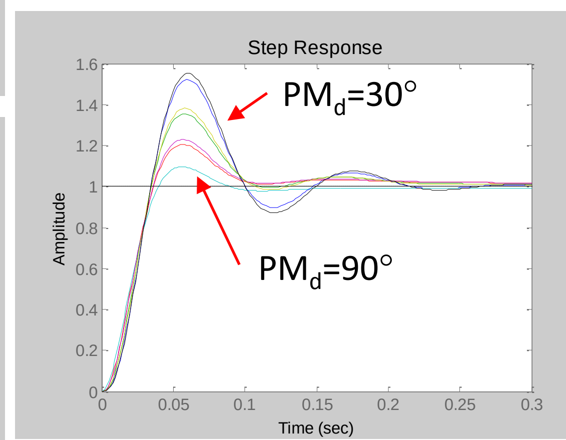
Matlab code:

```
close all; clear all; clc;
s=zpk('s'); G=100/(s*(s+1)*(0.0125*s+1));
wcd=50;  Ked=10;
for pmd=[30,40,50,60,70,80,90],
   Gc=leadlag(G,wcd,pmd,Ked);
   Go=Gc*G; Gcl=feedback(Go,1);
   figure(1), bode(Go); grid off; hold on;
   figure(2), nyquist(Go); grid off; hold on;
   figure(3), nichols(Go); grid off; hold on;
   figure(4), step(Gcl,0.3); grid off; hold on;
end
```









<u>**Note**</u>: Bad specs → Bad designs