Embedded Control and Mechatronics

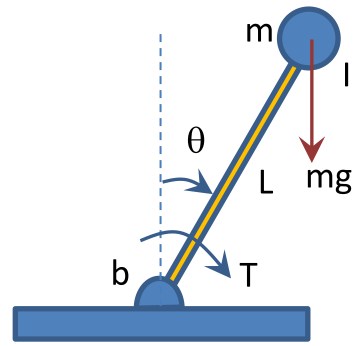
Spring 2012 — **ECE-456**

Dr. F. Pourboghrat — **Exam-2 (Take-Home)** — **Due: Wed, 05/09/2012**

(**Solve the problem, show simulation results and plots, provide the Matlab/Simulink programs**)

#1) Consider an upright pendulum, as in the figure, with dynamic model

$$\ddot{\theta} = \frac{1}{mL^2}T + \frac{g}{L}\,sin\theta - \frac{b}{mL^2}\dot{\theta} + d$$, where m=0.2, L=0.5, and b=0.2.



The objective is to develop a gain-scheduled controller to bring the pendulum from any initial position to its upright position.

a) Develop a **Simulink model** of the system, where disturbance $d = 0$.

b) Using Simulink, find the **equilibrium/operating point (op)** of the system at angle $\theta = 0$. Also generate the **Matlab code** that finds the operating point at the specified angle.

c) Using Simulink, find the **linearized state-space model** of the system at the above equilibrium-point $\begin{bmatrix}0\\0\end{bmatrix}$. Also generate the **Matlab code** that finds the linearized model of the system at the specified operating point.

d) For the linearized model at $\theta = 0$, write a Matlab code to design an **LQG controller** to move the pendulum from an initial condition $\begin{bmatrix}\theta(0)\\\dot{\theta}(0)\end{bmatrix} = \begin{bmatrix}\theta_0\\0\end{bmatrix}$ to the upright equilibrium point $\begin{bmatrix}0\\0\end{bmatrix}$, using weighting matrices $R = 1$ and $Q = 100\,I$ for the state-feedback gain design, and $\Theta = 1$ and $\Xi = 250\,I$ for the observer-gain design. Also, find the **range of the initial condition** $\begin{bmatrix}\theta_0\\0\end{bmatrix}$ that the designed controller could successfully move the pendulum to the upright equilibrium point $\begin{bmatrix}0\\0\end{bmatrix}$.

e) Using Matlab codes from parts (b), (c), and (d), write a **Matlab program** that finds the operating points (op) at $\theta = k\frac{\pi}{5}, k = 0, \pm 1, \ldots, \pm 5$, finds the linearized models of the system at these operating points, designs LQG controller for each of the linearized models, and sets up the necessary gains for the **gain-scheduled controller** to move the pendulum from any initial condition $\begin{bmatrix}\theta(0)\\\dot{\theta}(0)\end{bmatrix} = \begin{bmatrix}\theta_0\\0\end{bmatrix}$, with $-\pi \le \theta_0 \le \pi$, to the upright equilibrium position $\begin{bmatrix}0\\0\end{bmatrix}$.

f) **Draw the simulation plots** of the state, input, and output variables of the closed-loop system, when the above **gain-scheduled LQG controller** is applied to the Simulink model of the pendulum to move the pendulum from any initial position to its upright position.

g) For $d = \left(\sin(\dot{\theta})\,Cos(\theta)\right)^2$, **draw the simulation plots** of the state, input, and output variables of the closed-loop system, when the **gain-scheduled LQG controller** in part (f) is used to move the pendulum from any initial position to its upright position.

h) For $d = \left(\sin(\dot{\theta})\,Cos(\theta)\right)^2$, **repeat parts (d), (e), and (f)**, using an **H∞ controller design** with appropriate filters $W_i(s)$.