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Guidelines for Controller Designs Using State Space Method

Given a linear time-invariant (LTI) system: $\begin{cases} \dot{x} = F\,x + G\,u \\ y = H\,x + J\,u \end{cases}$,

State Space Control:

Case 1-
If reference command is, $r = 0$, and state vector x is directly measurable, the control, if the system is controllable, is $u = -K\,x$.

a. Choose the location of the desired closed-loop eigenvalues (poles), $s_1, s_2, \cdots s_n$, using the given specifications, and find the corresponding desired closed-loop characteristic equation, $\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n = (s - s_1)\cdots(s - s_n)$.
b. Check if the system is controllable.
c. Find the state feedback gain K from the equality: $a_c(s) = \det[sI - F + G\,K] = \alpha_c(s)$, or from Ackermann's formula: $K = [0,\cdots,0,1]C^{-1}\alpha_c(F)$, where C is the controllability matrix $C = [G, FG, \cdots, F^{n-1}G]$ and $\alpha_c(F) = F^n + \alpha_1 F^{n-1} + \cdots + \alpha_n I$.

Case 2-
If reference command is, $r \neq 0$, and state vector x is directly measurable, the control, if the system is controllable, is $u = -K\,x + N\,r$.

a. Find the state feedback gain K as in the above.
b. Find the feedforward gain N as: $N = N_u + K\,N_x$, where $\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} F & G \\ H & J \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Equivalently, $N = -\dfrac{1}{H\,(F - G\,K)^{-1} G}$.

Case 3-
If reference command is, $r = 0$, and state vector x is not directly measurable, the control, if the system is controllable and observable, is $\begin{cases} \dot{\hat{x}} = F\,\hat{x} + G\,u + L(y - H\,\hat{x}) \\ u = -K\,\hat{x} \end{cases}$.

a. Find the state feedback gain K as in the above.
b. Choose the location of the desired estimator (observer) eigenvalues, $s_{e1}, s_{e2}, \cdots s_{en}$, about 3 to 10 times to the left side of the dominant closed-loop poles, and find the desired estimator's characteristic equation, $\alpha_e(s) = s^n + \alpha_{e1} s^{n-1} + \cdots + \alpha_{en} = (s - s_{e1})\cdots(s - s_{en})$.
c. Check if the system is observable.
d. Find the state estimator (observer) gain L from equality $a_e(s) = \det[sI - F + L\,H] = \alpha_e(s)$, or from Ackermann's formula: $L = \alpha_c(F)\,O^{-1}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, where $O = \begin{bmatrix} H \\ H\,F \\ \vdots \\ H\,F^{n-1} \end{bmatrix}$ is the observability matrix and $\alpha_e(F) = F^n + \alpha_{e1} F^{n-1} + \cdots + \alpha_{en} I$.

Case 4-

If reference command is, $r \neq 0$, and state vector x <u>is not</u> directly measurable, the control, if the system is controllable and observable, is $\begin{cases} \dot{\hat{x}} = (F - G\,K - L\,H)\,\hat{x} + L\,y + M\,r \\ u = -K\,\hat{x} + N\,r \end{cases}$.

a. Find the state feedback gain K as in the above.
b. Find the state estimator (observer) gain L as in the above.
c. Find the feedforward gains M and N as follows:

<u>Autonomous estimator</u> (state estimator error equation is independent of r):

i. Choose N as in case 2.
ii. Choose $M = G\,N$.

<u>Tracking-error estimator</u> (only tracking error $e = r - y$ is used in the control):

i. Choose $N = 0$.
ii. Choose $M = -L$.

<u>Zero-assignment estimator</u> (n zeros of closed-loop control are assigned):

i. Choose the location of n transmission zeros, $z_1, z_2, \cdots z_n$ as:

$$\gamma(s) = s^n + \gamma_1 s^{n-1} + \cdots + \gamma_n = (s - z_1)\cdots(s - z_n).$$

ii. Find $\overline{M}$ from: $\det\left[sI - F + G\,K + L\,H - \overline{M}\,K\right] = \gamma(s)$.

iii. Choose $N = -\dfrac{1}{H\,(F - G\,K)^{-1}G\,[1 - K\,(F - L\,H)^{-1}(G - \overline{M})]}$.

iv. Choose $M = \overline{M}\,N$.

Integral Control:

To eliminate the steady state error due to constant disturbance w.

a) Introduce a new state variable x_I so that $\dot{x}_I = e = y - r = H\,x - r$.

b) Find the augmented system equation: $\begin{bmatrix} \dot{x}_I \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & H \\ 0 & F \end{bmatrix}\begin{bmatrix} x_I \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ G_1 \end{bmatrix} w$, where r is the reference signal and $x_a = \begin{bmatrix} x_I \\ x \end{bmatrix}$ is the augmented state vector. If the original system is controllable, the augmented system will also be controllable and the feedback control is:

$$u = -K_a\,x_a = -[K_I, K]\begin{bmatrix} x_I \\ x \end{bmatrix} = -K_I\,x_I + K\,x$$

c) Find the feedback control gain $K_a = [K_I, K]$ for the augmented system as in case1, described above.