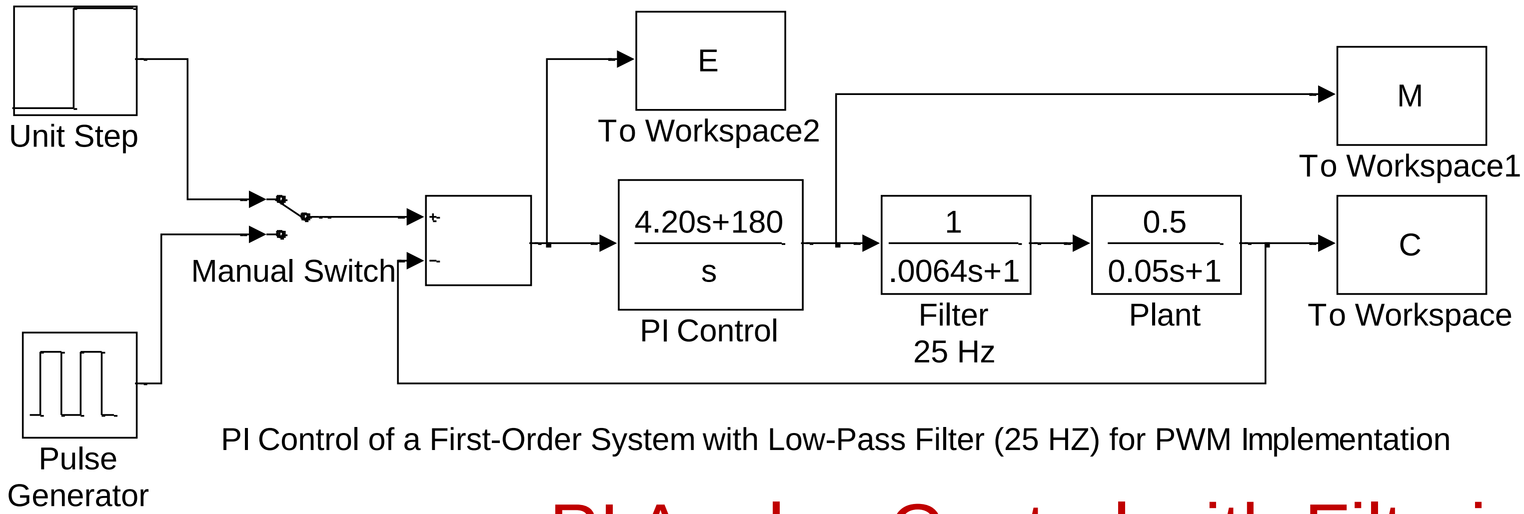
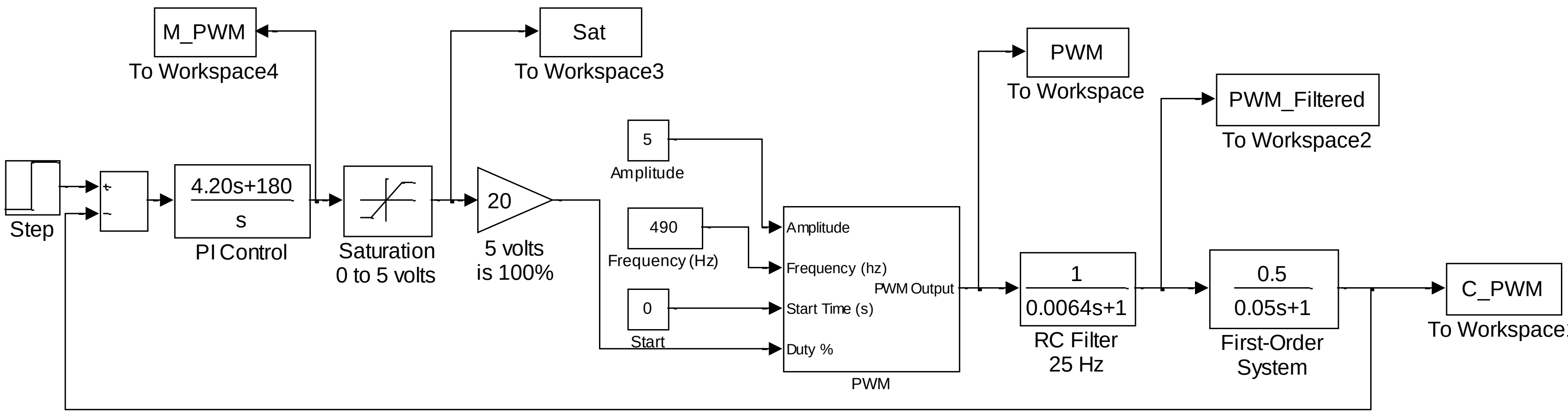


PI Control of a First-Order System with Low-Pass Filter (25 HZ) for PWM Implementation

PI Analog Control with Filtering

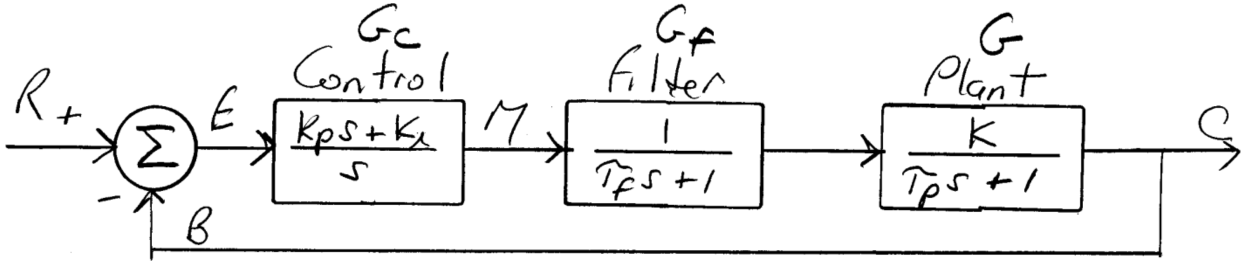


PI Control of a First-Order System with Low-Pass Filter (25 HZ) and PWM

PI Control – PWM w/ Saturation and Filtering

1^{st} Order Plant: PI Control with PWM

K. Craig [1]



The filter is used to convert the PWM control signal to an analog signal. Since this will be present in implementation, design the PI controller with the filter.

$$\frac{C}{R} = \frac{G_c G_f G}{1 + G_c G_f G}$$

$$G_c G_f G = \left(\frac{K_p s + K_i}{s}\right)\left(\frac{1}{\tau_f s + 1}\right)\left(\frac{K}{\tau_p s + 1}\right)$$

$$= \frac{K(K_p s + K_i)}{(\tau_f s + 1)(\tau_p s + 1)(s)}$$

$$= \frac{K(K_p s + K_i)}{\tau_f \tau_p s^3 + (\tau_p + \tau_f)s^2 + s}$$

$$\frac{C}{R} = \frac{K(K_p s + K_i)}{\tau_f \tau_p s^3 + (\tau_p + \tau_f)s^2 + (K K_p + 1)s + K K_i}$$

$$= \frac{\frac{K K_i}{\tau_f \tau_p}\left[\frac{K_p}{K_i} s + 1\right]}{s^3 + \left(\frac{1}{\tau_f} + \frac{1}{\tau_p}\right)s^2 + \left[\frac{K K_p + 1}{\tau_f \tau_p}\right]s + \frac{K K_i}{\tau_f \tau_p}}$$

Let denominator $= (s + p)(s^2 + 2\zeta\omega_n s + \omega_n^2)$

$$\frac{C}{R} = \frac{p\omega_n^2(\tau s + 1)}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{p\omega_n^2(\tau s+1)}{s^3 + (2\zeta\omega_n + p)s^2 + (2\zeta\omega_n p + \omega_n^2)s + p\omega_n^2}$$

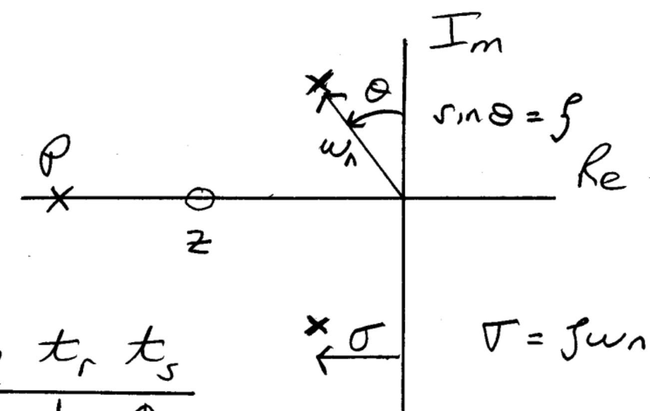
$$\left(\frac{1}{\tau_f} + \frac{1}{\tau_p}\right) = 2\zeta\omega_n + p$$

Given $\begin{cases} \tau_f = .0064 \\ \tau_p = .05 \\ K = 0.5 \end{cases}$

$$\frac{KK_p + 1}{\tau_f \tau_p} = 2\zeta\omega_n p + \omega_n^2 \implies K_p = \frac{(\tau_f \tau_p)(2\zeta\omega_n p + \omega_n^2) - 1}{K}$$

$$\frac{KK_i}{\tau_f \tau_p} = p\omega_n^2 \implies K_i = \frac{p\omega_n^2 \tau_f \tau_p}{K}$$

$$\tau = \frac{K_p}{K_i} \qquad z = \frac{1}{\tau}$$



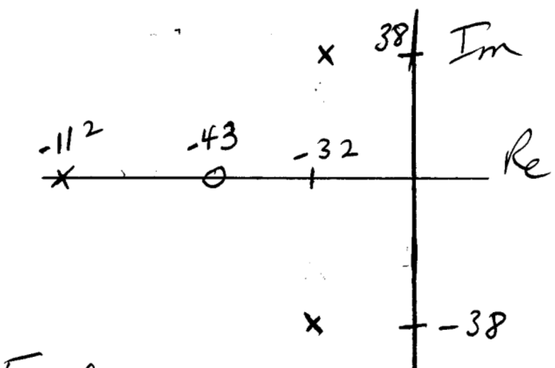
pole-zero plot

	M_p	t_r	t_s
As $z \rightarrow$	↑	↓	↑
As $p \rightarrow$	↓	↑	↑

Choose: $\zeta = 0.64$ $\quad \omega_n = 50 \quad \Rightarrow \quad \sigma = \zeta \omega_n = 32$ $\quad \boxed{3}$

Then: $p = 112.25$ $\qquad \dfrac{p}{\sigma} = 3.5$

$$\left.\begin{aligned} K_p &= 4.20 \\ K_i &= 180 \end{aligned}\right\} \qquad z = \frac{K_i}{K_p} = 42.9 \qquad \frac{z}{\sigma} = 1.34$$



<u>Performance</u>

$t_{r_{10-90\%}} = .025$ sec $\quad > .015$ sec

$M_p = 17.5\%$ $\quad < 25\%$

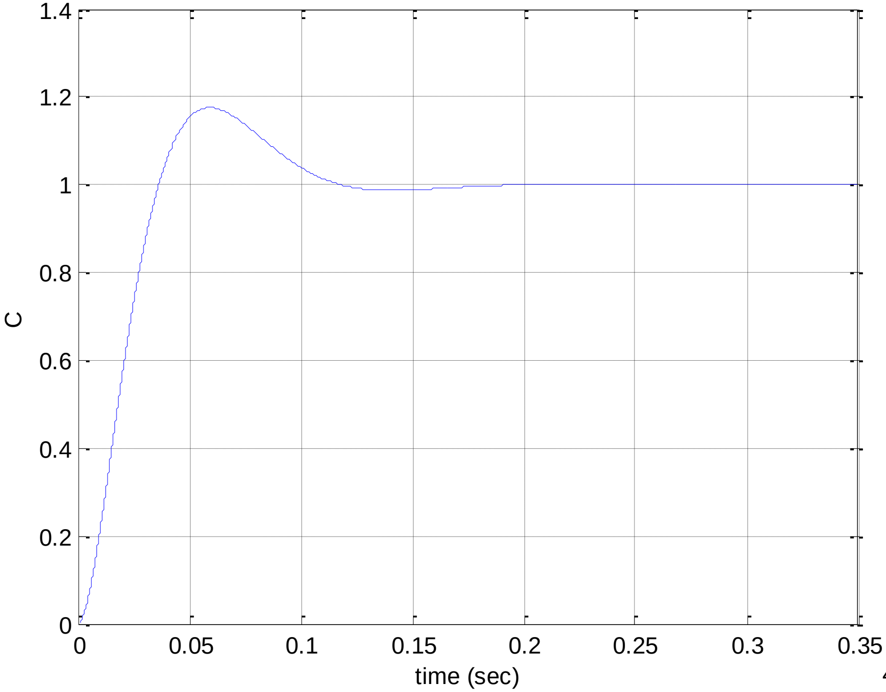
$t_p = .060$ sec

$t_{s_{1\%}} = 0.157$ sec $\quad > .09$ sec

Desired

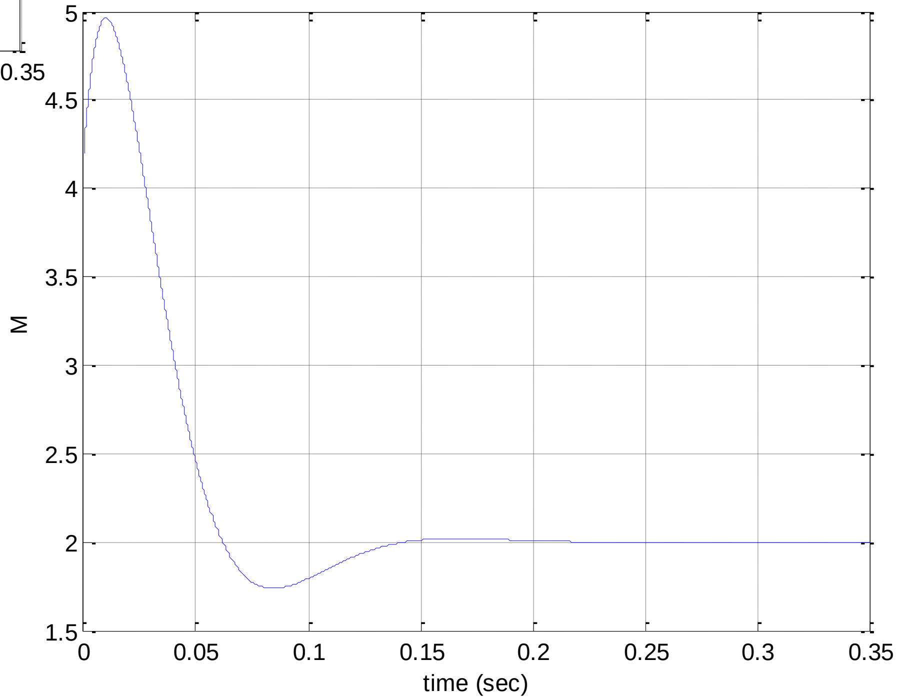
<u>No Saturation</u>

$M < 5$ volts



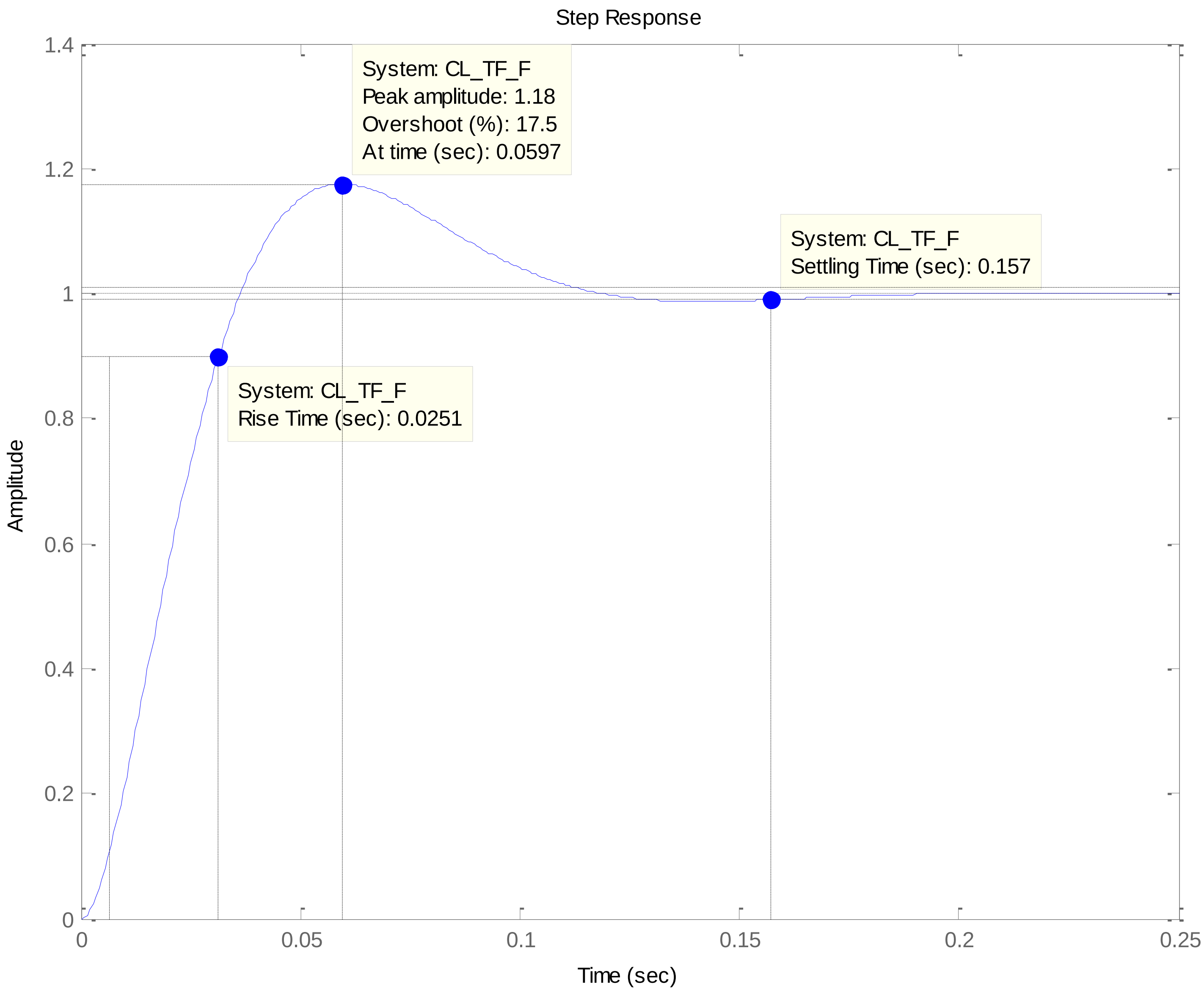
Controlled Variable C

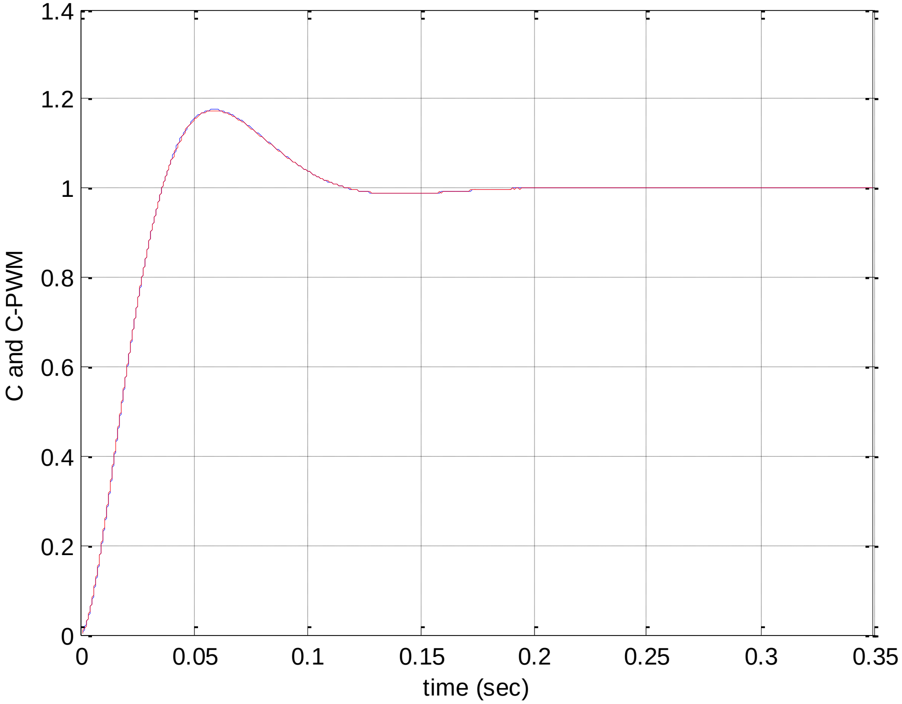
Control Effort M



PI Analog Control with Filtering

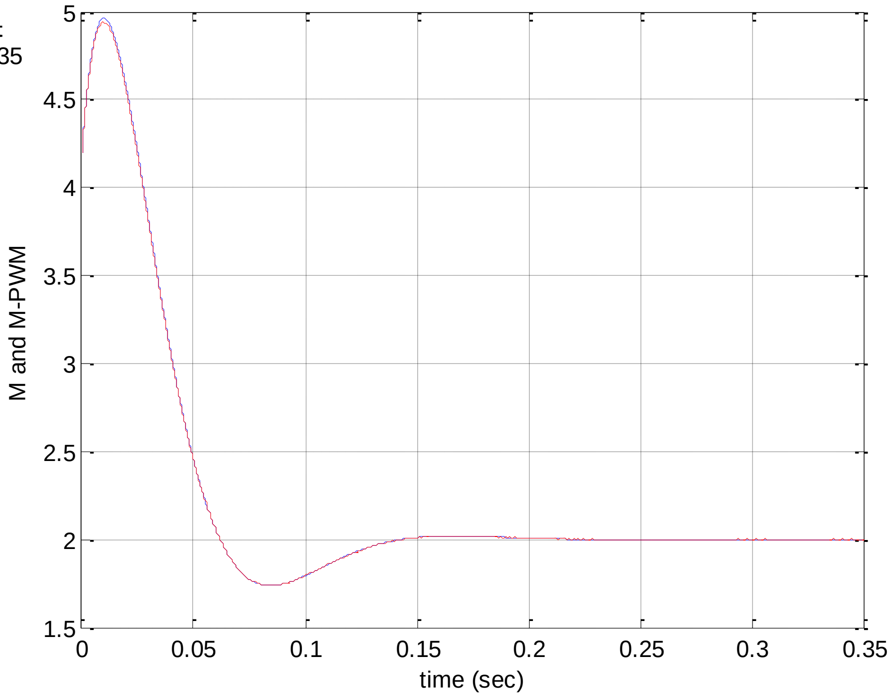
PI Analog Control with Filtering





Controlled Variable C
C and C-PWM are Identical

Control Effort M
M and M-PWM are Identical



PI Analog Control with Filtering and PWM

M-PWM and Filtered-PWM