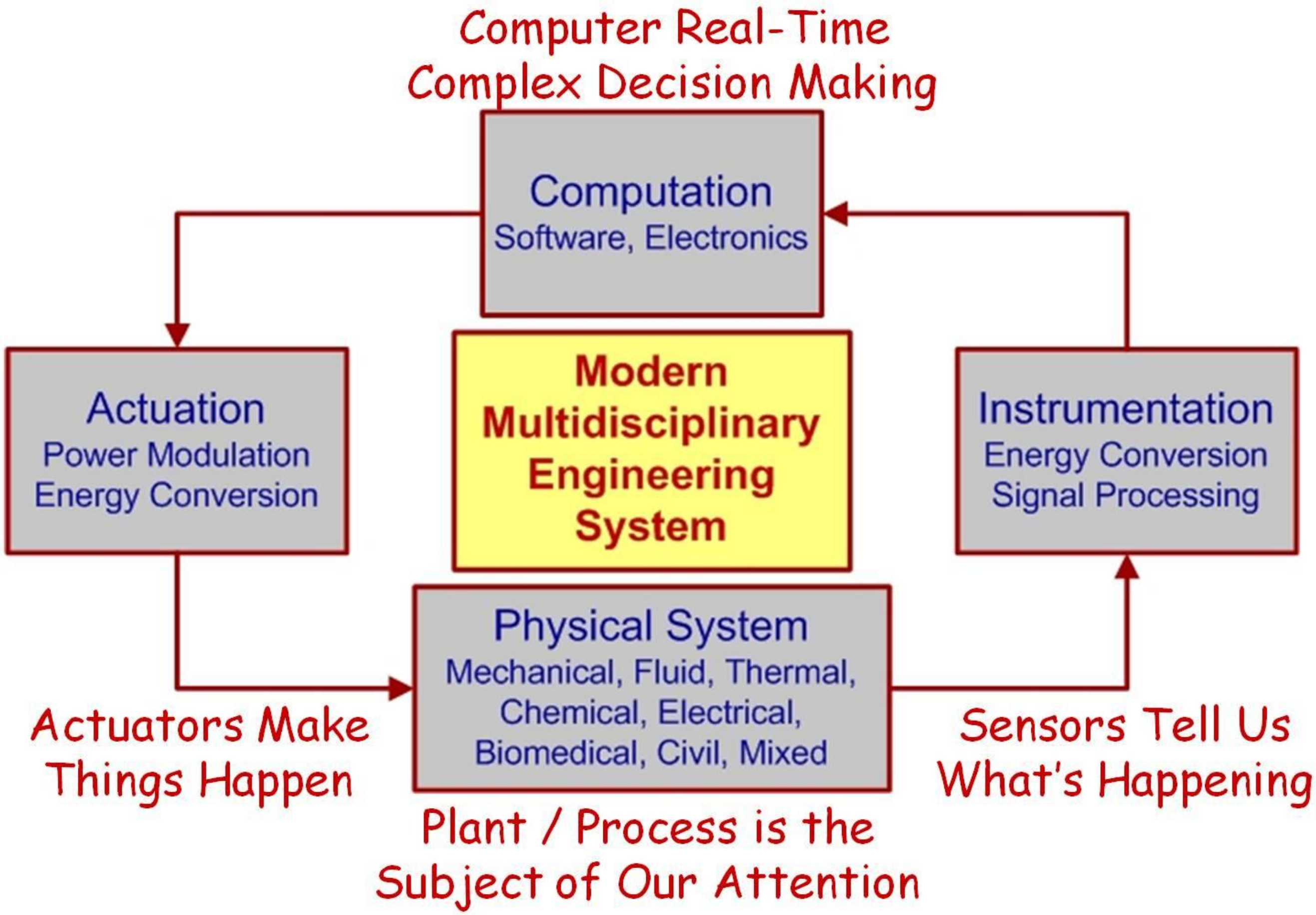
Feedback Control Systems

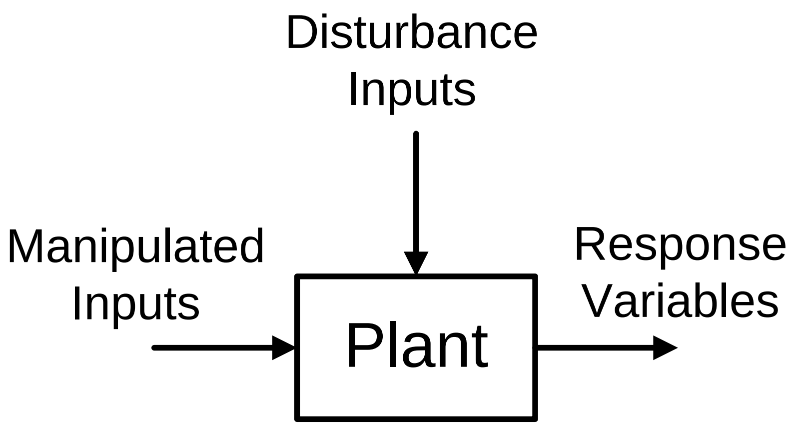


Feedback Control Topics

- Part 1
  - Control System Types
    - Open-Loop Control: Basic & Feedforward
    - Closed-Loop Control
  - Block Diagrams and Loading Effects
  - Generalized Feedback Control Block Diagram
  - Feedback Control Transfer Functions
  - Sensitivity of Control Systems to Parameter Variation
  - Negative Feedback and Op-Amps
  - Instability in Feedback Control Systems

Control System Types

- Process or Plant
- Process Inputs
  - Manipulated Inputs
  - Disturbance Inputs
- Response Variables



Control systems are an integral part of the overall system and not after-thought add-ons!

The earlier the issues of control are introduced into the design process, the better!

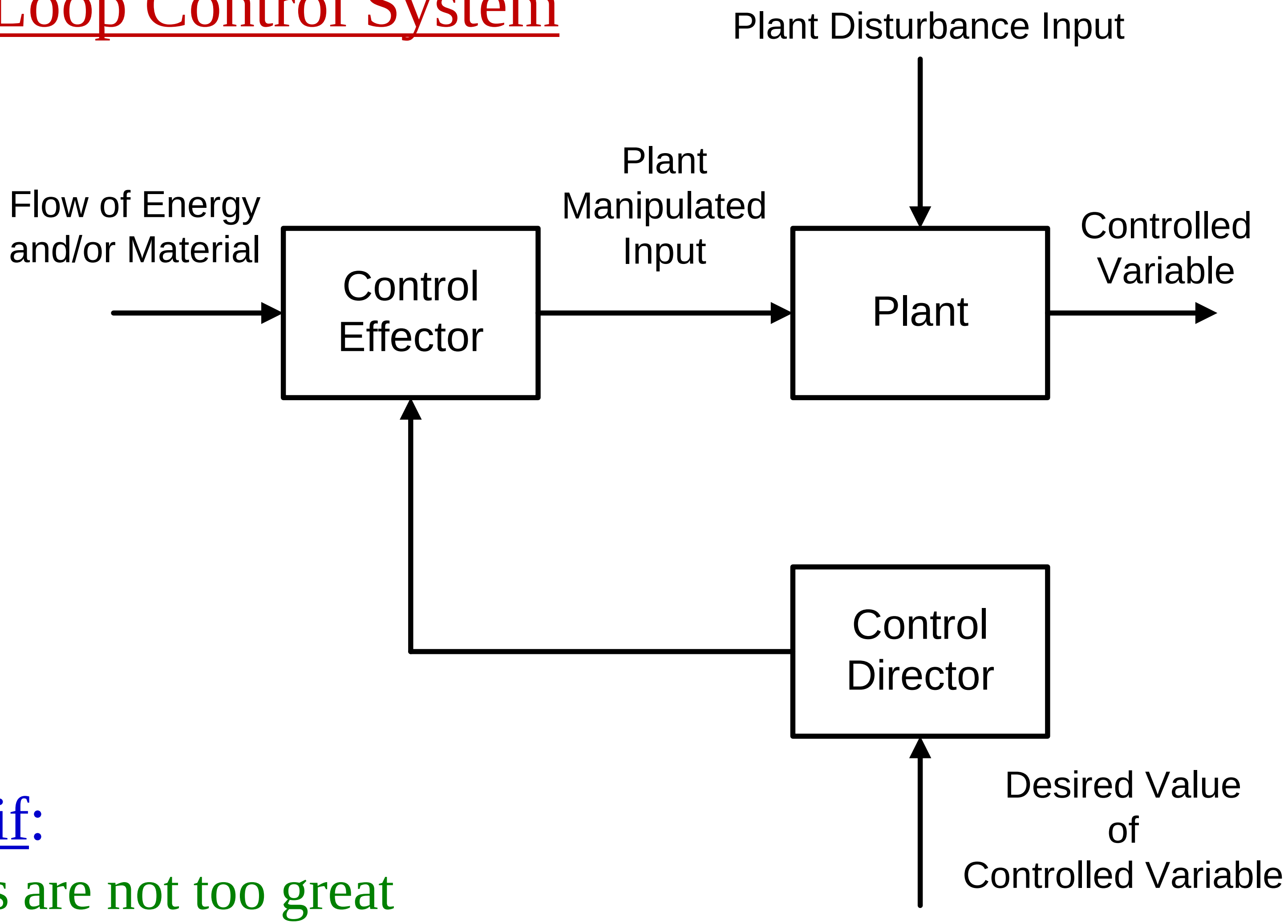
Why Controls?

- Command Following
- Disturbance Rejection
- Parameter Variations

Everything Needs Controls for Optimum Functioning!

- Classification of Control System Types
  - Open-Loop
    - Basic
    - Input-Compensated Feedforward
      - Disturbance-Compensated
      - Command-Compensated
  - Closed-Loop (Feedback)
    - Classical (e.g., PID)
      - Root-Locus
      - Frequency Response
    - Modern (State-Space)
    - Advanced
      - e.g., Adaptive, Fuzzy Logic

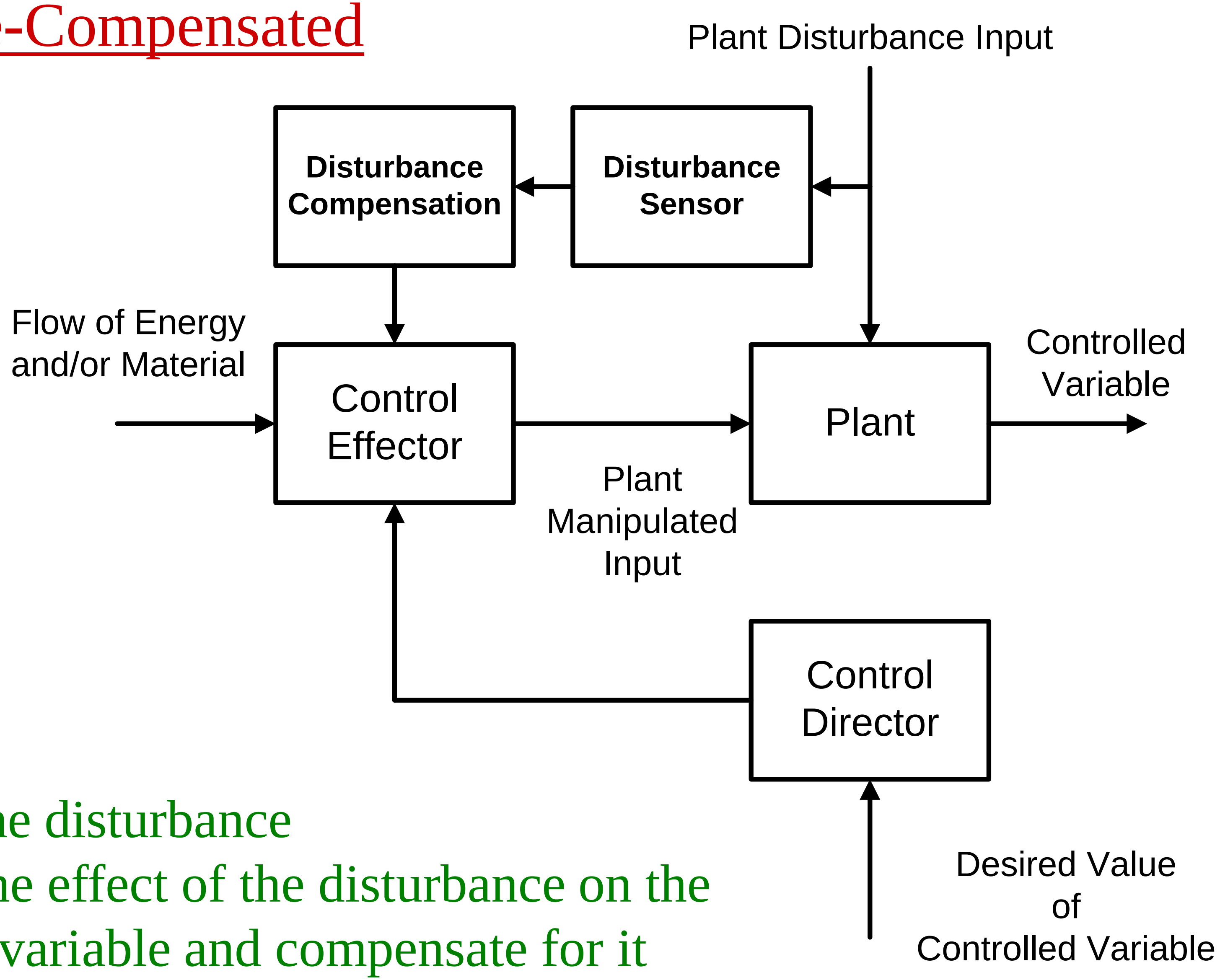
Basic Open-Loop Control System



Satisfactory if:

- disturbances are not too great
- changes in the desire value are not too severe
- performance specifications are not too stringent

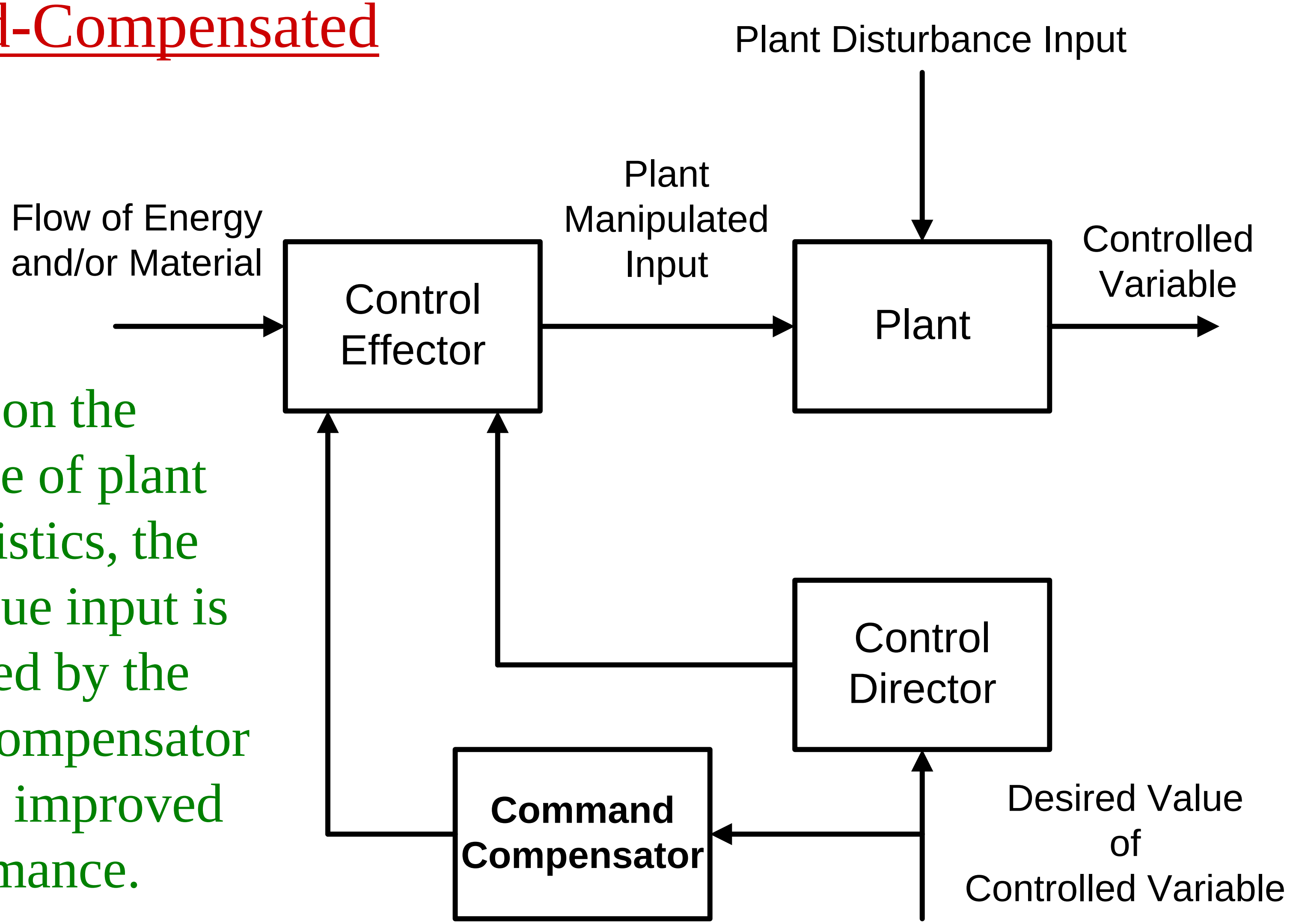
Open-Loop Input-Compensated Feedforward Control: Disturbance-Compensated



- Measure the disturbance
- Estimate the effect of the disturbance on the controlled variable and compensate for it

- Disturbance-Compensated Feedforward Control
  - Basic Idea: Measure important load variables and take corrective action before they upset the process.
  - In contrast, a feedback controller, as we will see, does not take corrective action until after the disturbance has upset the process and generated an error signal.
  - There are several disadvantages to disturbance-compensated feedforward control:
    - The load disturbances must be measured on line. In many applications, this is not feasible.
    - The quality of the feedforward control depends on the accuracy of the process model; one needs to know how the controlled variable responds to changes in both the load and manipulated variables.
    - Ideal feedforward controllers that are theoretically capable of achieving perfect control may not be physically realizable. Fortunately, practical approximations of these ideal controllers often provide very effective control.

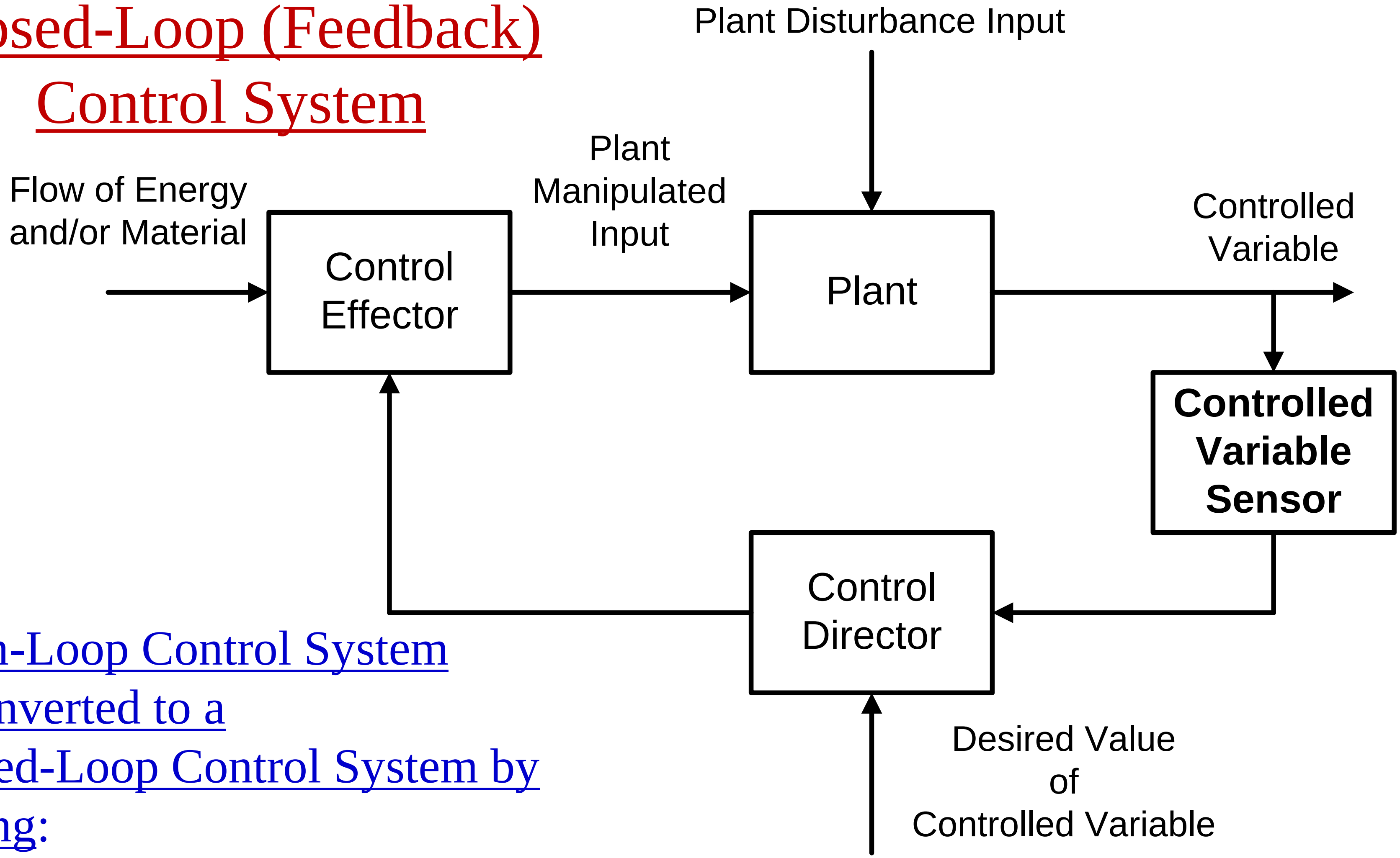
Open-Loop Input-Compensated Feedforward Control: Command-Compensated



Based on the knowledge of plant characteristics, the desired value input is augmented by the command compensator to produce improved performance.

- Comments:
  - Open-loop systems without disturbance or command compensation are generally the simplest, cheapest, and most reliable control schemes. These should be considered first for any control task.
  - If specifications cannot be met, disturbance and/or command compensation should be considered next.
  - When conscientious implementation of open-loop techniques by a knowledgeable designer fails to yield a workable solution, the more powerful feedback methods should be considered.

Closed-Loop (Feedback) Control System



Open-Loop Control System is converted to a Closed-Loop Control System by adding:

- measurement of the controlled variable
- comparison of the measured and desired values of the controlled variable

- Basic Benefits of Feedback Control
  - Cause the controlled variable to accurately follow the desired variable; corrective action occurs as soon as the controlled variable deviates from the command.
  - Greatly reduces the effect on the controlled variable of all external disturbances in the forward path. It is ineffective in reducing the effect of disturbances in the feedback path (e.g., those associated with the sensor), and disturbances outside the loop (e.g., those associated with the reference input element).
  - Is tolerant of variations (due to wear, aging, environmental effects, etc.) in hardware parameters of components in the forward path, but not those in the feedback path (e.g., sensor) or outside the loop (e.g., reference input element).

- Can give a closed-loop response speed much greater than that of the components from which they are constructed.

- <u>Inherent Disadvantages of Feedback Control</u>
  - No corrective action is taken until after a deviation in the controlled variable occurs. Thus, perfect control, where the controlled variable does not deviate from the set point during load or set-point changes, is theoretically impossible.
  - It does not provide predictive control action to compensate for the effects of known or measurable disturbances.

- It may not be satisfactory for processes with large time constants and/or long time delays. If large and frequent disturbances occur, the process may operate continually in a transient state and never attain the desired steady state.
- In some applications, the controlled variable cannot be measured on line and, consequently, feedback control is not feasible.

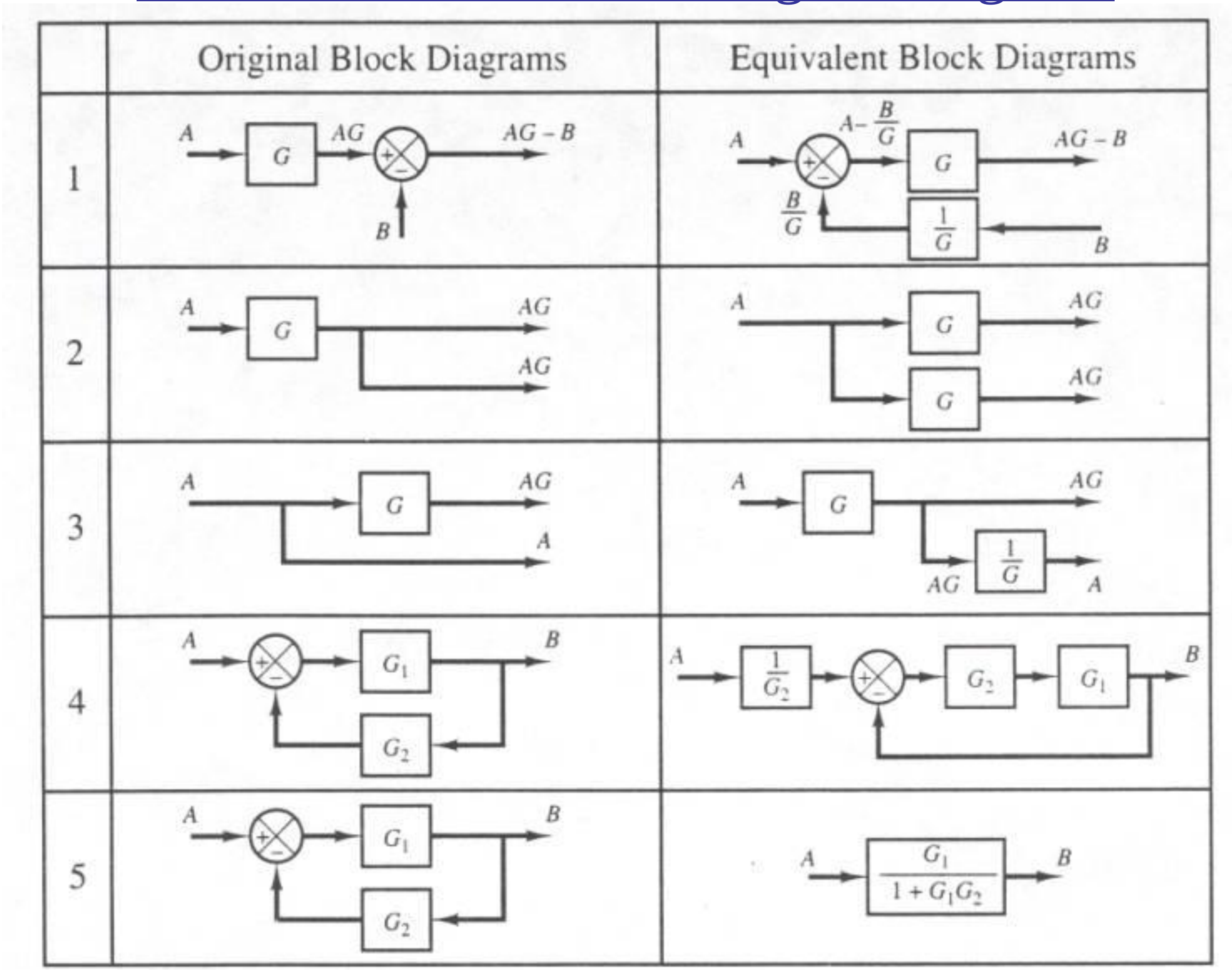
• For situations in which feedback control by itself is not satisfactory, significant improvements in control can be achieved by adding feedforward control.

Block Diagrams & Loading Effects

- A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. It depicts the interrelationships that exist among the various components.
- It is easy to form the overall block diagram for the entire system by merely connecting the blocks of the components according to the signal flow. It is then possible to evaluate the contribution of each component to the overall system performance.
- A block diagram contains information concerning dynamic behavior, but it does not include any information on the physical construction of the system.

- Many dissimilar and unrelated systems can be represented by the same block diagram.
- A block diagram of a given system is not unique. A number of different block diagrams can be drawn for a system, depending on the point of view of the analysis.
- Blocks can be connected in series only if the output of one block is not affected by the next following block. If there are any loading effects between components, it is necessary to combine these components into a single block.

Some Rules of Block Diagram Algebra



Loading Effects

- The unloaded transfer function is an incomplete component description.
- To properly account for interconnection effects one must know three component characteristics:
  - the unloaded transfer function of the upstream component
  - the output impedance of the upstream component
  - the input impedance of the downstream component
- Only when the ratio of output impedance over input impedance is small compared to 1.0, *over the frequency range of interest*, does the unloaded transfer function give an accurate description of interconnected system behavior.

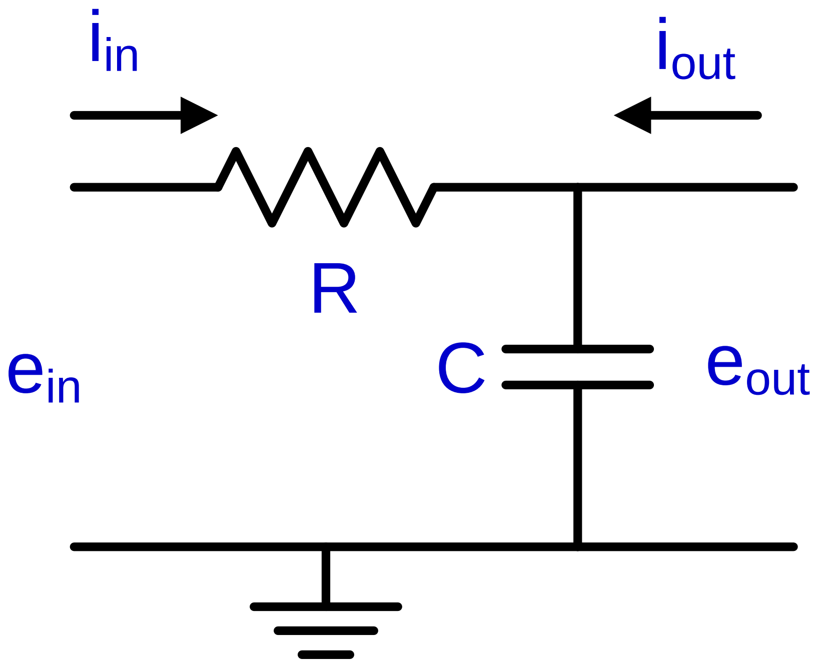
$$u \longrightarrow \boxed{G_1} \longrightarrow \boxed{G_2} \longrightarrow y$$

$$\frac{Y(D)}{U(D)} = \left[G_1(D) \frac{1}{1 + \dfrac{Z_{o1}}{Z_{i2}}} \right] G_2(D)$$

Only if $\dfrac{Z_{o1}}{Z_{i2}} << 1$ for the <u>frequency range of interest</u> will loading effects be negligible.

- In general, loading effects occur because when analyzing an isolated component (one with no other component connected at its output), we assume no power is being drawn at this output location.
- When we later decide to attach another component to the output of the first, this second component does withdraw some power, violating our earlier assumption and thereby invalidating the analysis (transfer function) based on this assumption.
- When we model chains of components by simple multiplication of their individual transfer functions, we assume that loading effects are either not present, have been proven negligible, or have been made negligible by the use of buffer amplifiers.

RC Low-Pass Filter



$$\text{KCL} \Rightarrow i_{in} + i_{out} - C\dot{e}_{out} = 0$$

$$i_{in} = -i_{out} + CDe_{out}$$

$$\text{KVL} \Rightarrow e_{in} - Ri_{in} - e_{out} = 0$$

$$e_{in} = e_{out} + Ri_{in}$$

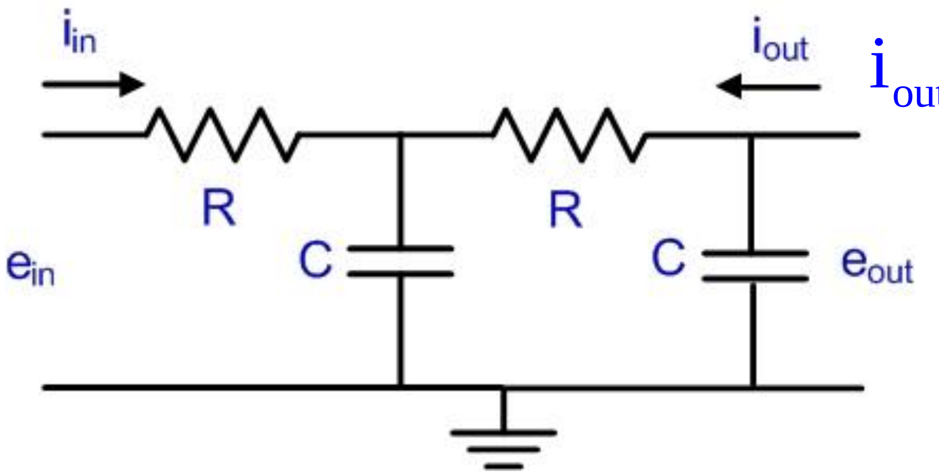
$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} RCD+1 & -R \\ CD & -1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix}$$

$$\left[\frac{e_{out}}{e_{in}}\right]_{i_{out}=0} = \frac{1}{RCD+1}$$

Ideal Transfer Function

Unloaded since $i_{out} = 0$

Two RC Low-Pass Filters in Series



$$i_{out} = 0$$

NO !

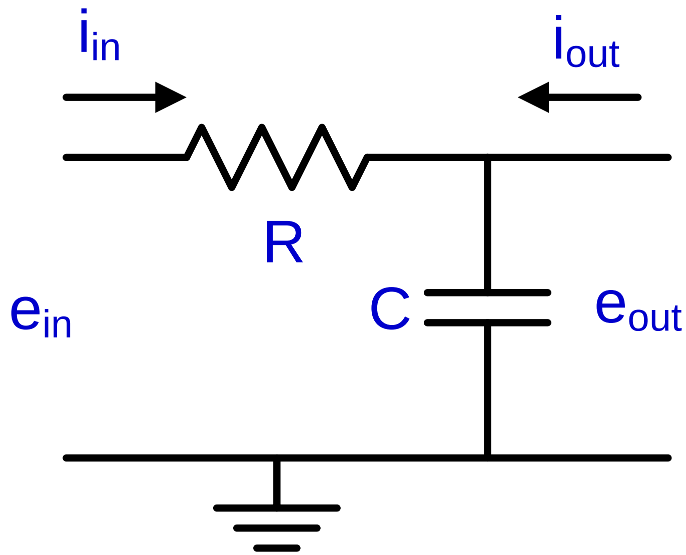
$$e_{in} \rightarrow \boxed{\left(\frac{1}{RCD+1}\right)^2} \rightarrow e_{out}$$

$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} RCD+1 & -R \\ CD & -1 \end{bmatrix} \begin{bmatrix} RCD+1 & -R \\ CD & -1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix}$$

$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} R^2C^2D^2 + 3RCD + 1 & -R^2CD - 2R \\ RC^2D^2 + 2CD & -RCD - 1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix}$$

$$e_{in} \rightarrow \boxed{\frac{1}{R^2C^2D^2 + 3RCD + 1}} \rightarrow e_{out}$$

Loading-Effects Example
RC Low-Pass Filter

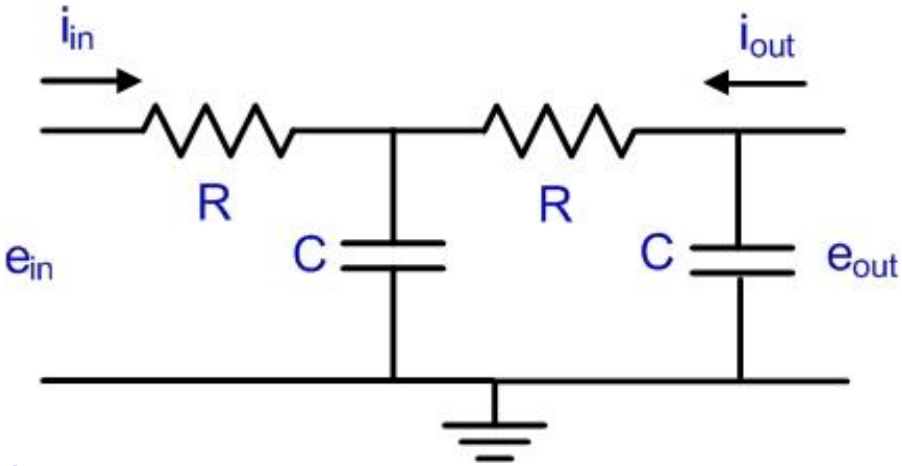


$$\begin{bmatrix} e_{in} \\ i_{in} \end{bmatrix} = \begin{bmatrix} RCD+1 & -R \\ CD & -1 \end{bmatrix} \begin{bmatrix} e_{out} \\ i_{out} \end{bmatrix}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{RCD+1} = \frac{1}{\tau D+1} \qquad \text{when } i_{out} = 0$$

$$\Rightarrow \quad Z_{out} = \left.\frac{e_{out}}{i_{out}}\right|_{e_{in}=0} = \frac{R}{RCD+1} \qquad \text{Output Impedance}$$

$$\Rightarrow \quad Z_{in} = \left.\frac{e_{in}}{i_{in}}\right|_{i_{out}=0} = \frac{RCD+1}{CD} \qquad \text{Input Impedance}$$



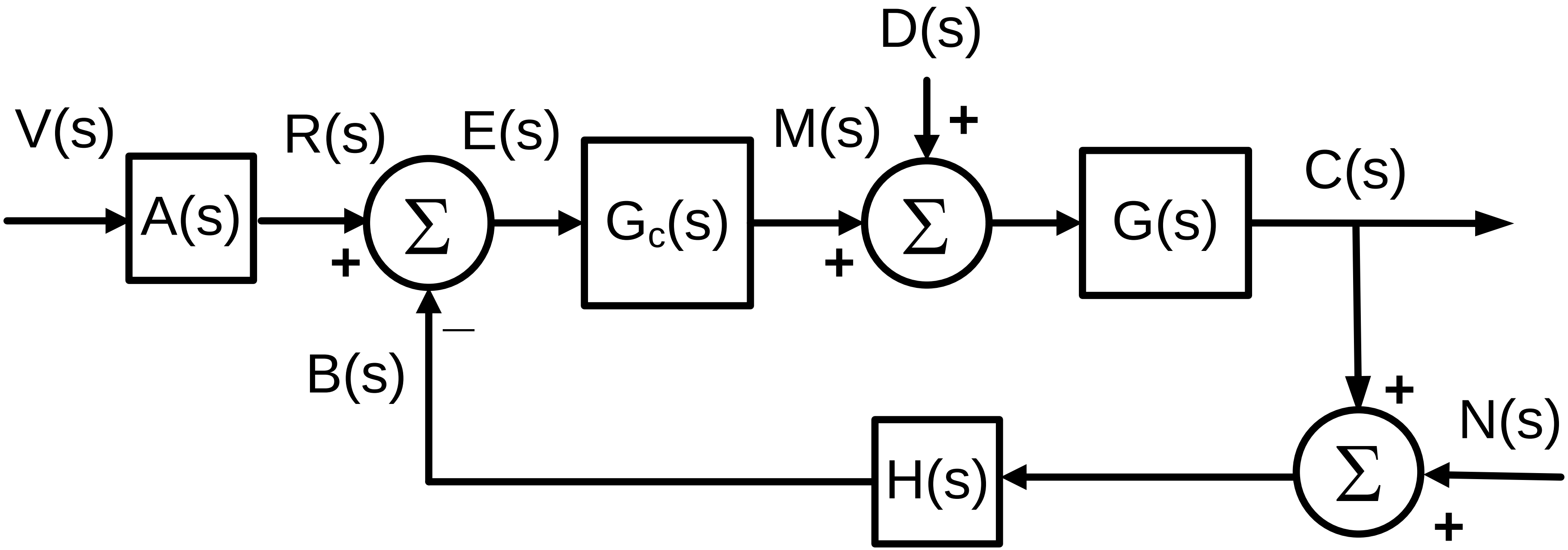
Two RC Low-Pass Filters in Series

$$\frac{e_{out}}{e_{in}} \neq G(D)_{1-unloaded} G(D)_{2-unloaded} = \left(\frac{1}{RCD+1}\right)\left(\frac{1}{RCD+1}\right)$$

$$\frac{e_{out}}{e_{in}} = G(D)_{1-loaded} G(D)_{2-unloaded} = \left(\frac{1}{RCD+1}\right)\left(\frac{1}{1+\frac{Z_{out-1}}{Z_{in-2}}}\right)\left(\frac{1}{RCD+1}\right)$$

$$= \frac{1}{(RCD+1)^2 + RCD}$$

Only if $Z_{out\text{-}1} << Z_{in\text{-}2}$
for the frequency range of interest
will loading effects be negligible.



Feedback (Closed-Loop) Control System
Generalized Block Diagram

- Earlier block diagrams have been of a general functional nature. Now it is appropriate to begin using the working operational block diagrams necessary for actual system design and analysis.
- These use the transfer function concept which allows the block diagram to communicate the numerical details of component and system behavior. The figure identifies the basic functional components from which all feedback systems are built. Two types of quantities require definition: *signals* and *systems*.
  - Signals are the physical variables (e.g., voltage, pressure, temperature, etc.) that "flow" from one system component to another.
  - Systems are the hardware components that perform the necessary operations. System descriptions consist of the transfer functions A(s), G(s), etc., which are shorthand graphic means of stating the component's differential equation.

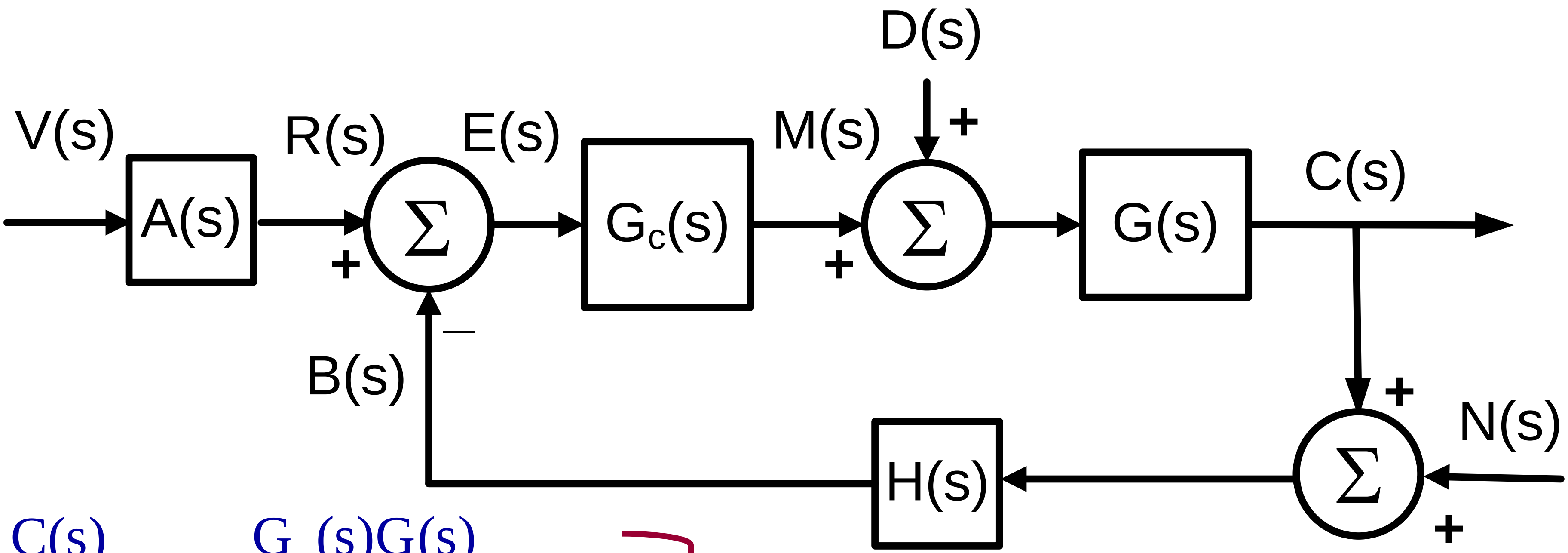
- Signal V is the desired value of the controlled variable C with the same units as C. V may or may not exist as an actual physical quantity.
- The fact that signals V and R, the reference input element, need not be the same quantity is one reason the standard diagram provides the transfer function A(s), the reference input element. The reference input element can, when necessary, perform a simple function, e.g., an algebraic conversion, or a more sophisticated function, e.g., command compensation or noise filtering.
- The summing junction represents the comparison, $E = R - B$, of the reference input with the feedback signal B. The summing junction cannot change the units of R and B, therefore R, B, and E always have exactly the same dimensions.

- The feedback element H(s) is often a sensor for measuring C. But its functions sometimes include more than simple measurement, therefore feedback element is used rather than sensing element for its name.
- System error is logically defined as V - C, therefore actuating signal is a more appropriate term for E since E = V - C only if A(s) = H(s) = 1.0, which is sometimes true, but often not true.
- $G_c(s)$ represents the control elements, containing the functions of both controller and actuator.
- G(s) represents the controlled system elements or the process/plant to be controlled.

- Controlled variable C is influenced by both manipulated variable M and disturbance D. Since the effect on C of D and M would in general be different, the path from D to C is often provided with a disturbance input element transfer function (not shown), allowing completely independent specification of C/D and C/M relationships. This transfer function is not system components intentionally added by the designer to allow the disturbance entry to the system, but rather the necessary modeling of the unavoidable effect of D on C. The same applies to the sensor disturbance input N(s).

- The figure defines the basic types of signals and components necessary for description of any feedback control system. However, it must be adapted to the needs of each specific design. For example, disturbances may enter the system at several locations, not just at the process or sensor. This is easily accommodated by providing suitable located summing junctions and defining disturbances with corresponding disturbance input elements.
- Finally, when we deal with a specific application, rather than the abstract generality of the figure, it is preferable to use standard signals as subscripts on symbols which relate more directly to the physical variables involved, e.g., desired temperature value, T_V, and controlled variable, T_C.

Feedback Control System Block Diagram



$$\frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}$$

$$\frac{C(s)}{D(s)} = \frac{G(s)}{1+G_c(s)G(s)H(s)}$$

$$\frac{C(s)}{N(s)} = \frac{-G_c(s)G(s)H(s)}{1+G_c(s)G(s)H(s)}$$

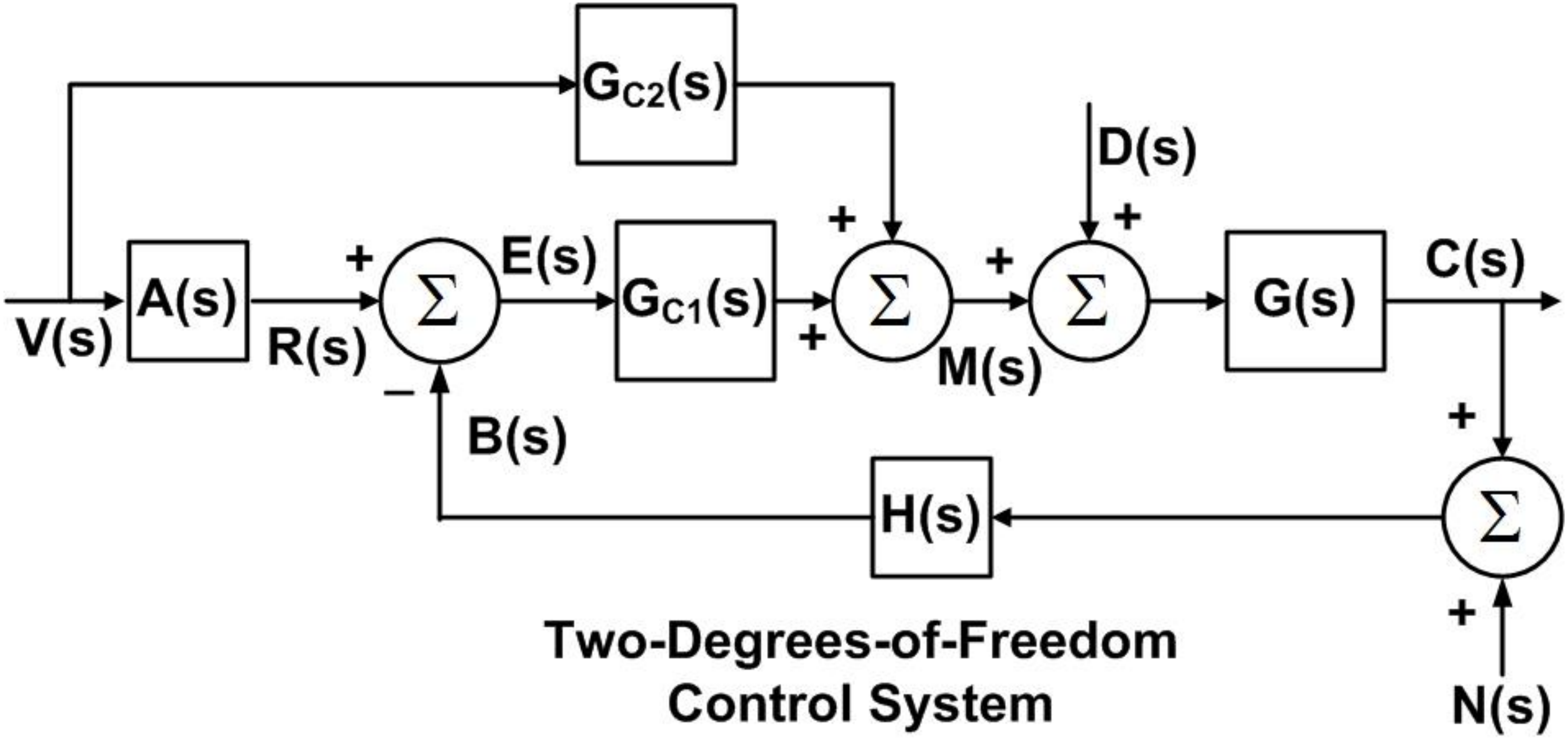
Closed Loop

$$\frac{B(s)}{E(s)} = G_c(s)G(s)H(s)$$

Open Loop

$$\frac{C(s)}{E(s)} = G_c(s)G(s)$$

Feedforward



Two-Degrees-of-Freedom
Control System

Sensitivity Analysis

- Consider the function y = f(x). If the parameter *x* changes by an amount Δx, then *y* changes by the amount Δy. If Δx is small, Δy can be estimated from the slope dy/dx as follows:

$$\Delta y = \frac{dy}{dx}\Delta x$$

- The relative or percent change in *y* is Δy/y. It is related to the relative change in *x* as follows:

$$\frac{\Delta y}{y} = \frac{dy}{dx}\frac{\Delta x}{y} = \left(\frac{x}{y}\frac{dy}{dx}\right)\frac{\Delta x}{x}$$

- The sensitivity of *y* with respect to changes in *x* is given by:

$$S_x^y = \frac{x}{y}\frac{dy}{dx} = \frac{dy/y}{dx/x} = \frac{d(\ln y)}{d(\ln x)}$$

- Thus

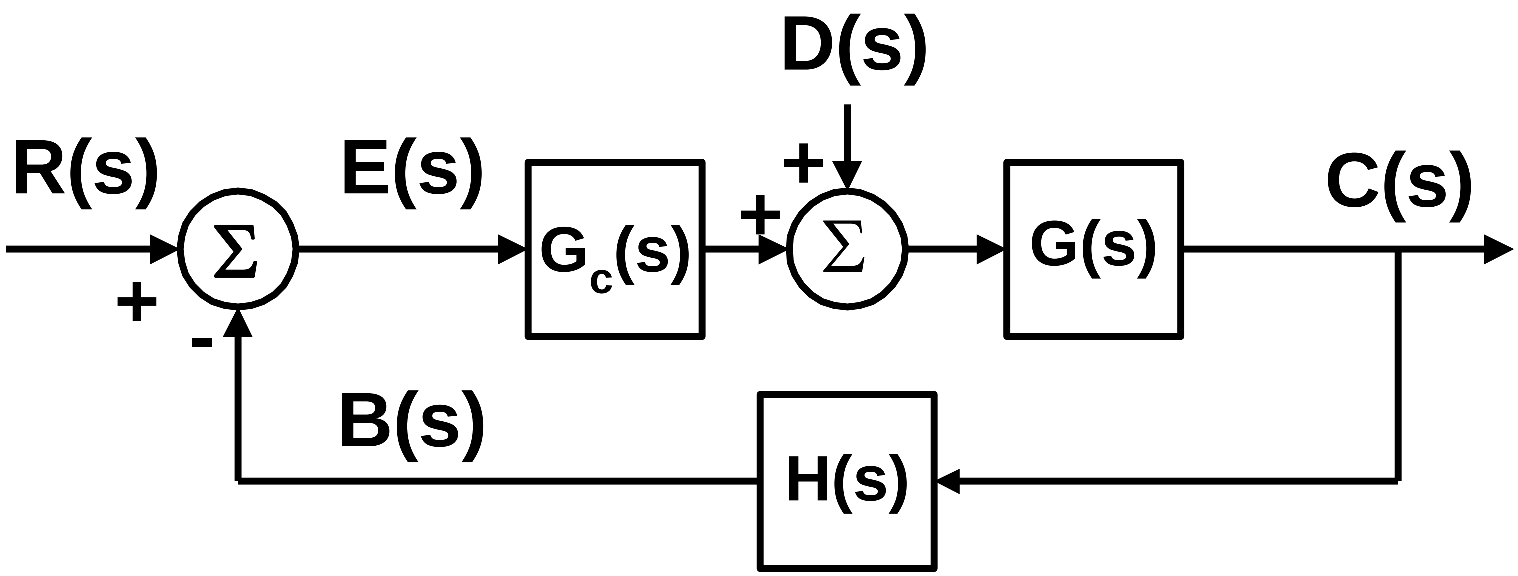
$$\frac{\Delta y}{y} = S_x^y \frac{\Delta x}{x}$$

- Usually the sensitivity is not constant. For example, the function y = sin(x) has the sensitivity function:

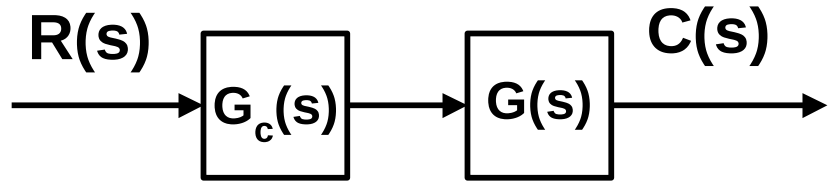
$$S_x^y = \frac{x}{y}\frac{dy}{dx} = \frac{x}{y}\cos(x) = \frac{x\cos(x)}{\sin(x)} = \frac{x}{\tan(x)}$$

- <u>Sensitivity of Control Systems to Parameter Variation and Parameter Uncertainty</u>
  - A process, represented by the transfer function G(s), is subject to a changing environment, aging, ignorance of the exact values of the process parameters, and other natural factors that affect a control process.
  - In the open-loop system, all these errors and changes result in a changing and inaccurate output.
  - However, a closed-loop system senses the change in the output due to the process changes and attempts to correct the output.
  - The sensitivity of a control system to parameter variations is of prime importance.

– Accuracy of a measurement system is affected by parameter changes in the control system components and by the influence of external disturbances.
– A primary advantage of a closed-loop feedback control system is its ability to reduce the system's sensitivity.
– Consider the closed-loop system shown. Let the disturbance D(s) = 0.



– An open-loop system's block diagram is given by:



– The system sensitivity is defined as the ratio of the percentage change in the system transfer function T(s) to the percentage change in the process transfer function G(s) (or parameter) for a small incremental change:

$$T(s) = \frac{C(s)}{R(s)}$$

$$S_G^T = \frac{\partial T / T}{\partial G / G} = \frac{\partial T}{\partial G}\frac{G}{T}$$

– For the open-loop system

$$T(s) = \frac{C(s)}{R(s)} = G_c(s)G(s)$$

$$S_G^T = \frac{\partial T / T}{\partial G / G} = \frac{\partial T}{\partial G}\frac{G}{T} = G_c(s)\frac{G(s)}{G_c(s)G(s)} = 1$$

– For the closed-loop system

$$T(s) = \frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}$$

$$S_G^T = \frac{\partial T / T}{\partial G / G} = \frac{\partial T}{\partial G}\frac{G}{T}$$

$$= \frac{1}{(1+G_cGH)^2}\frac{G}{\dfrac{G_cG}{1+G_cGH}} = \frac{1}{G_c\left(1+G_cGH\right)}$$

- The sensitivity of the system may be reduced below that of the open-loop system by increasing $G_cGH(s)$ over the frequency range of interest.
- The sensitivity of the closed-loop system to changes in the feedback element H(s) is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)}$$

$$S_H^T = \frac{\partial T / T}{\partial H / H} = \frac{\partial T}{\partial H}\frac{H}{T}$$

$$= \frac{-(G_cG)^2}{(1+G_cGH)^2}\frac{H}{\dfrac{G_cG}{1+G_cGH}} = \frac{-G_cGH}{\left(1+G_cGH\right)}$$

- When G_cGH is large, the sensitivity approaches unity and the changes in H(s) directly affect the output response. Use feedback components that will not vary with environmental changes or can be maintained constant.
- As the gain of the loop (G_cGH) is increased, the sensitivity of the control system to changes in the plant and controller decreases, but the sensitivity to changes in the feedback system (measurement system) becomes -1.
- Also the effect of the disturbance input can be reduced by increasing the gain G_cH since:

$$C(s) = \frac{G(s)}{1 + G_c(s)G(s)H(s)} D(s)$$

- Therefore:
  - Make the measurement system very accurate and stable.
  - Increase the loop gain to reduce sensitivity of the control system to changes in plant and controller.
  - Increase gain G_cH to reduce the influence of external disturbances.
- In practice:
  - G is usually fixed and cannot be altered.
  - H is essentially fixed once an accurate measurement system is chosen.
  - Most of the design freedom is available with respect to G_c only.

- It is virtually impossible to achieve all the design requirements simply by increasing the gain of G_c. The dynamics of G_c also have to be properly designed in order to obtain the desired performance of the control system.
- Very often we seek to determine the sensitivity of the closed-loop system to changes in a parameter α within the transfer function of the system G(s). Using the chain rule we find:

$$S_{\alpha}^{T} = S_{G}^{T} S_{\alpha}^{G}$$

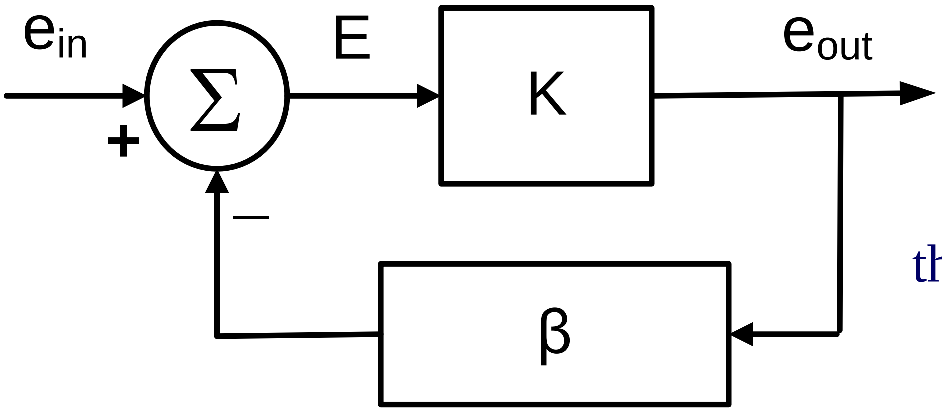
- Very often the transfer function T(s) is a fraction of the form:

$$T(s,\alpha) = \frac{N(s,\alpha)}{D(s,\alpha)}$$

– Then the sensitivity to α (α_0 is the nominal value) is given by:

$$S_{\alpha}^{T} = \frac{\partial T / T}{\partial \alpha / \alpha} = \frac{\partial \ln T}{\partial \ln \alpha} = \left.\frac{\partial \ln N}{\partial \ln \alpha}\right|_{\alpha_0} - \left.\frac{\partial \ln D}{\partial \ln \alpha}\right|_{\alpha_0} = S_{\alpha}^{N} - S_{\alpha}^{D}$$

Negative Feedback and Op-Amps



βK is called the <u>Loop Gain</u>.

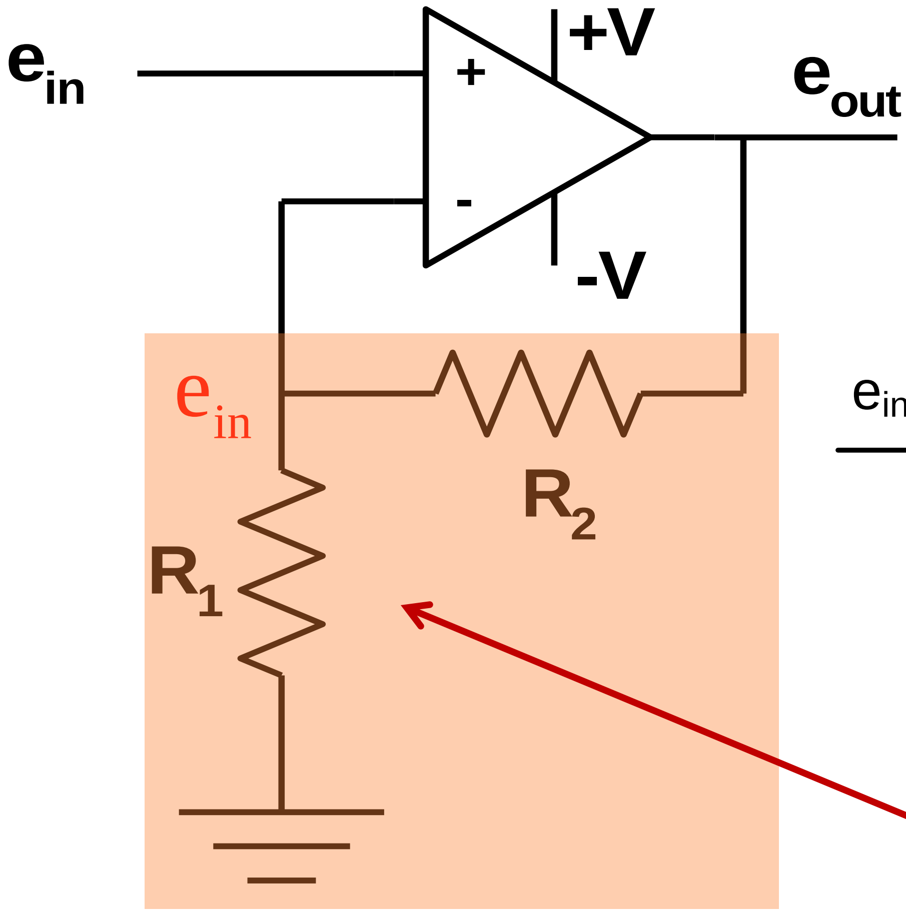
$$\frac{e_{out}}{e_{in}} = \frac{K}{1+\beta K}$$

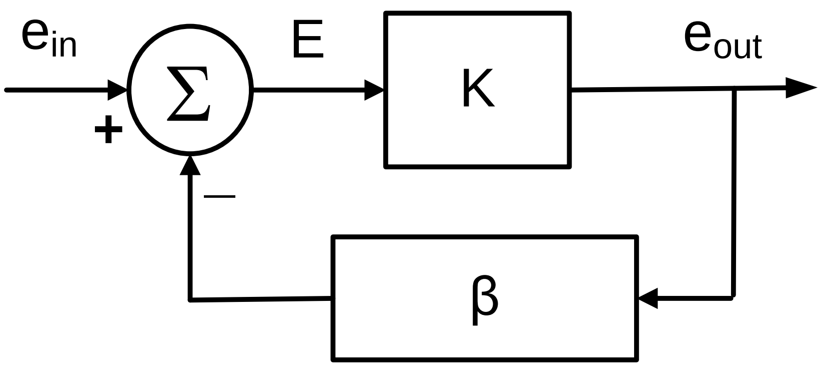
$$\frac{e_{out}}{e_{in}} = \frac{1}{\beta} \qquad \beta K >> 1$$

Negative feedback is the process of coupling the output back in such a way as to cancel some of the input. This does lower the amplifier's gain, but in exchange it also improves other characteristics, most notably freedom from distortion and nonlinearity, flatness of response (or conformity to some desired frequency response), and predictability. In fact, as more negative feedback is used, the resultant amplifier characteristics become less dependent on the characteristics of the open-loop (no feedback) amplifier and finally depend only on the properties of the feedback network itself. Operational amplifiers are typically used in this high-loop-gain limit, with open-loop voltage gain (no feedback) of a million or so.

Non-Inverting Op-Amp

$$\frac{e_{out}}{e_{in}} = 1 + \frac{R_2}{R_1}$$





$$\beta = \frac{R_1}{R_1 + R_2} = \frac{e_{in}}{e_{out}}$$

Voltage Divider

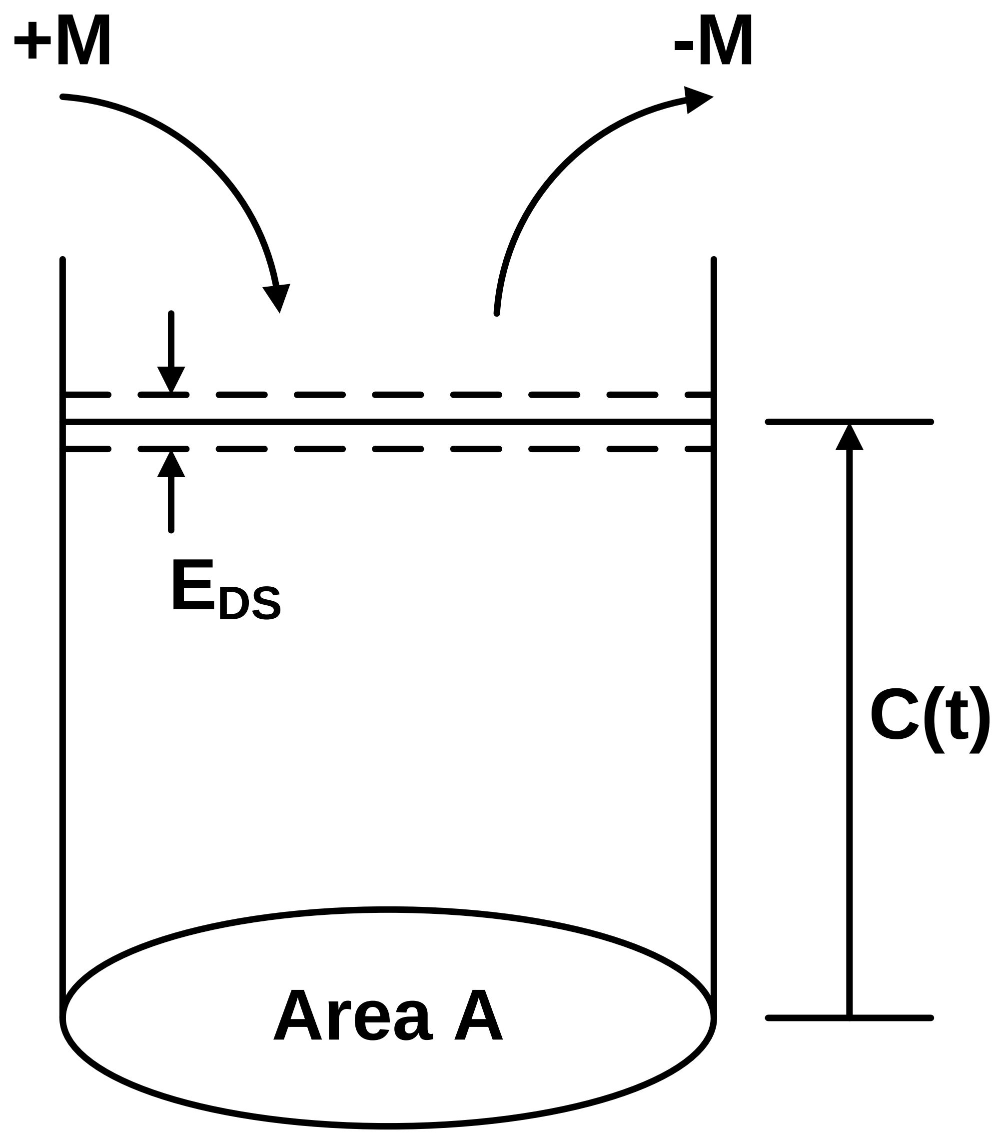
$$\frac{e_{out}}{e_{in}} = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Instability in Feedback Control Systems

- All feedback systems can become unstable if improperly designed.
- In all real-world components there is some kind of lagging behavior between the input and output.
- Instantaneous response is impossible in the real world!
- *Instability in a feedback control system results from an improper balance between the strength of the corrective action and the system dynamic lags.*

Example

- Liquid level *C* in a tank is manipulated by controlling the volume flow rate *M* by means of a three-position on/off controller with error dead space E_{DS}.
- Transfer function $1/As$ between *M* and *C* represents conservation of volume between volume flow rate and liquid level.
- Liquid-level sensor measures *C* perfectly but with a data transmission delay τ_{dt}.

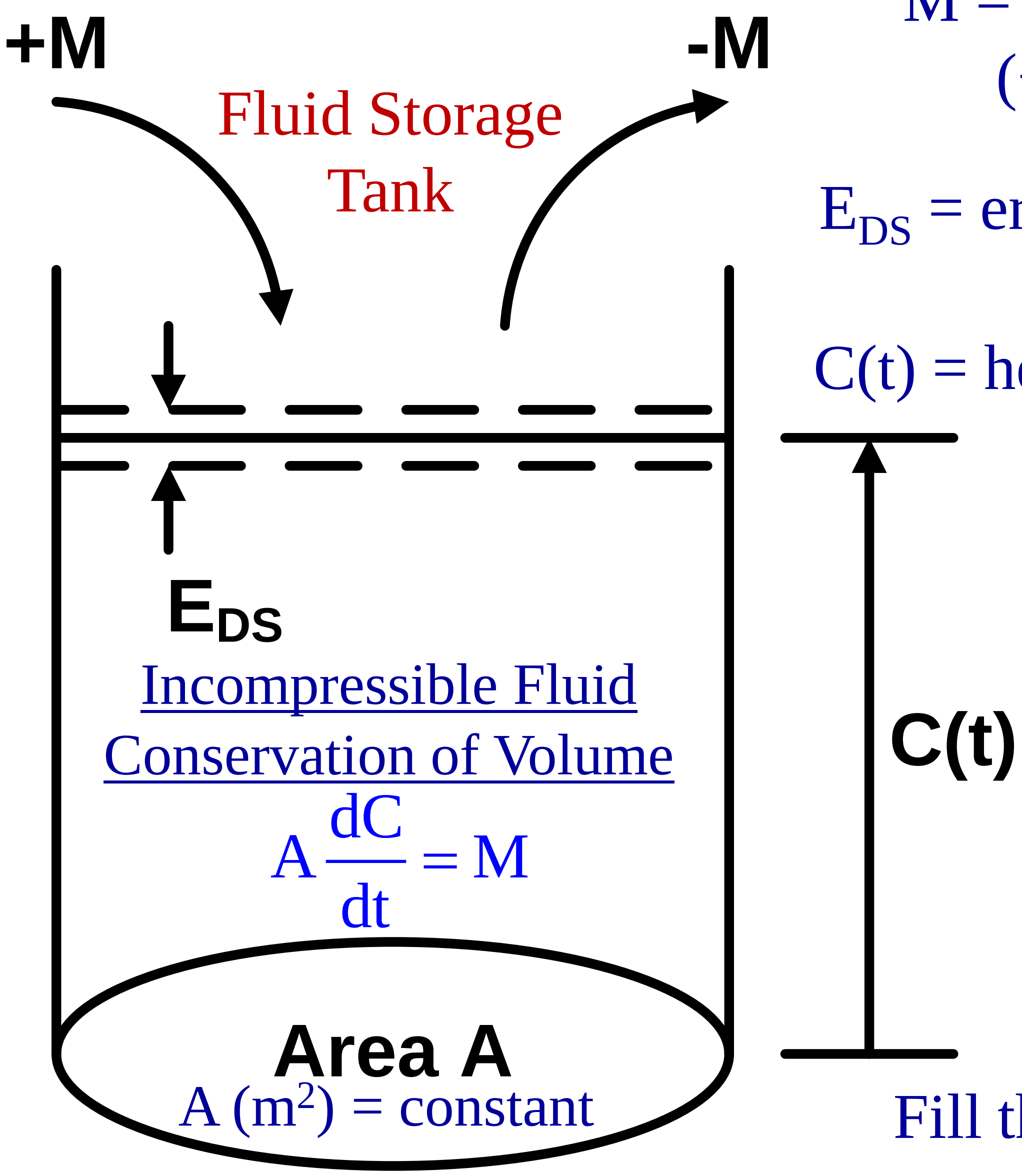


Tank Liquid-Level Feedback Control System

M = Volume Flow Rate (m^3/s)
(+M, -M, or 0 are possible)

E_{DS} = error dead space = 0.2 m

C(t) = height of fluid in tank (m)



Incompressible Fluid

Conservation of Volume

$$A\frac{dC}{dt} = M$$

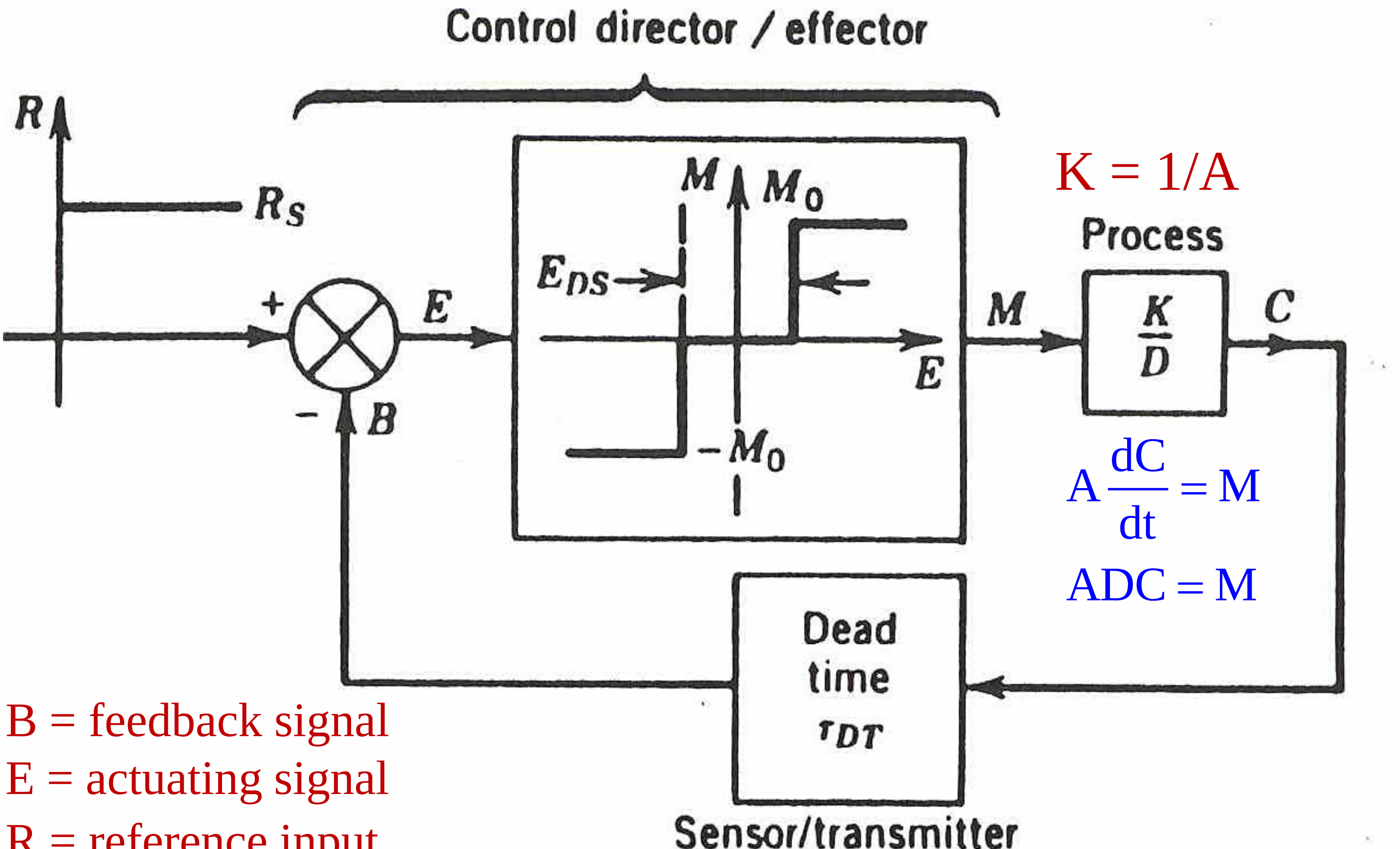
A (m^2) = constant

τ_{DT}

There is a time delay (seconds) in transmitting the fluid level measurement to the controller.

Objective

Fill the tank to the desired level $C \pm \frac{1}{2} E_{DS}$ and stop.



$K = 1/A$

$$A\frac{dC}{dt} = M$$

$$ADC = M$$

B = feedback signal

E = actuating signal

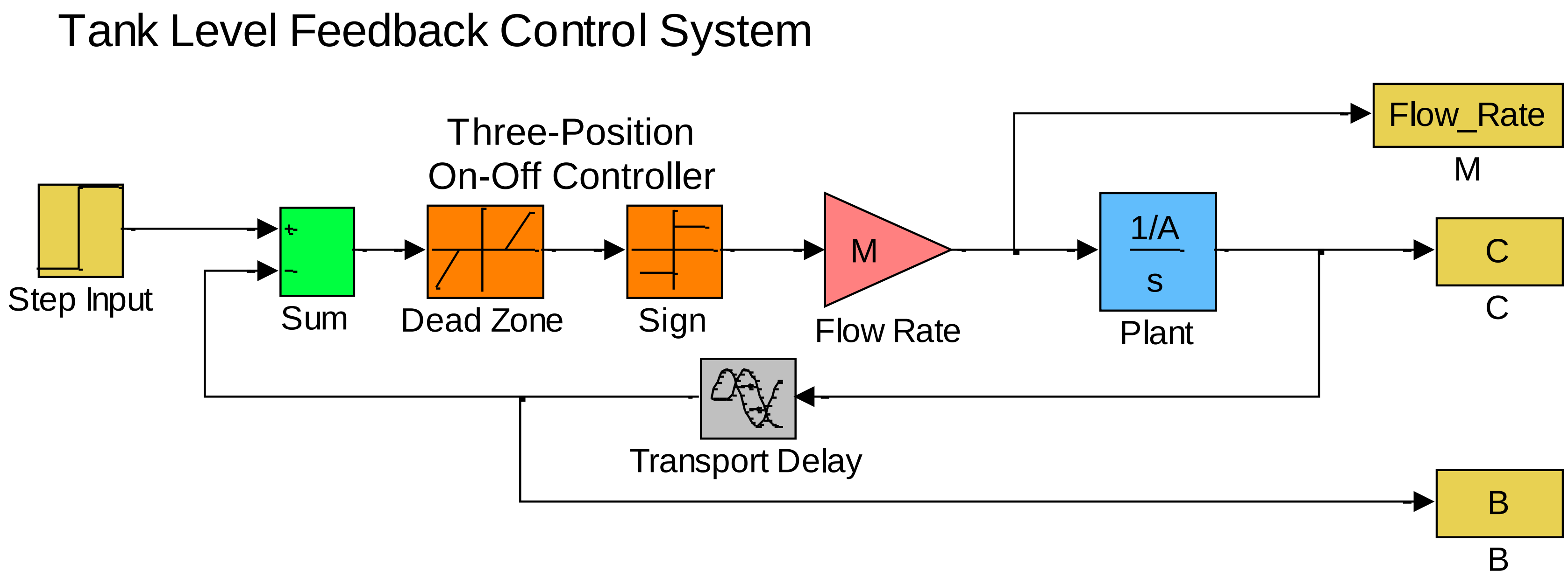
R = reference input

R_S = step reference input

M = manipulated input

C = controlled variable

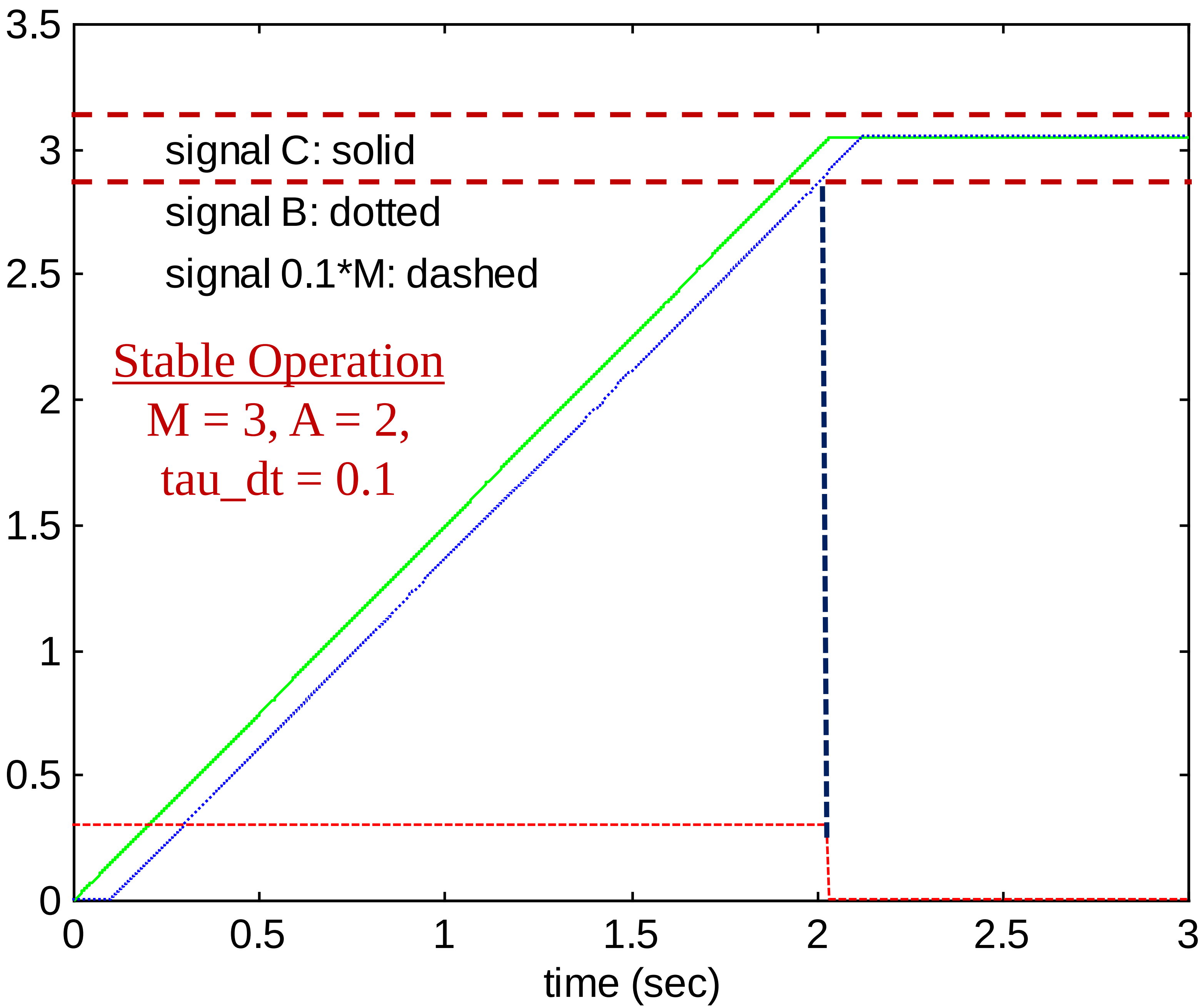
MatLab / Simulink Block Diagram



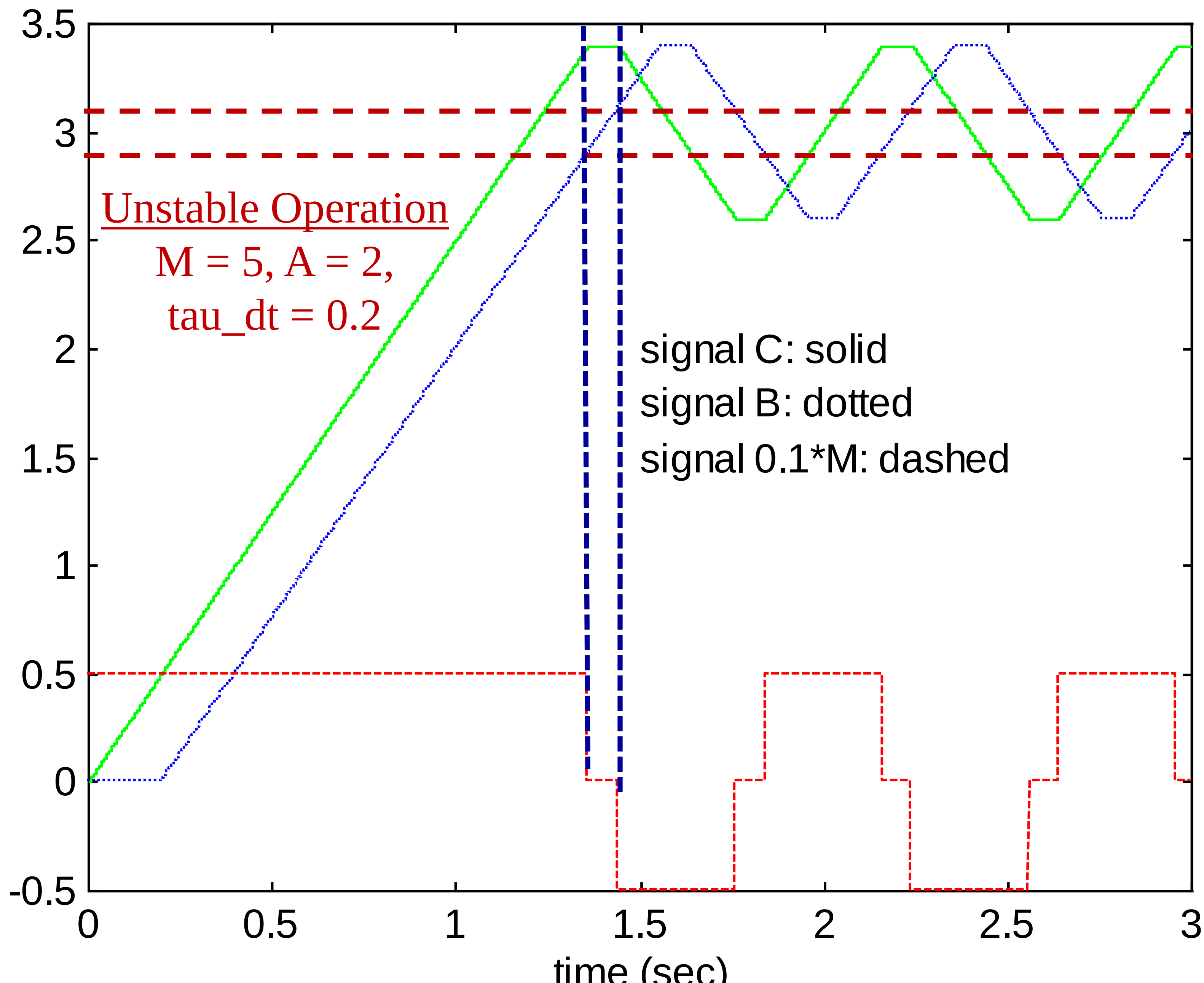
M = 3, A = 2, tau_dt = 0.1 : stable

M = 5, A = 2, tau_dt = 0.2 : unstable

Instability in a feedback control system results from an improper balance between the strength of the corrective action (here the combination of M and 1/A) and the system dynamic lags (here the transport delay).



signal C: solid
signal B: dotted
signal 0.1*M: dashed
Stable Operation
M = 3, A = 2,
tau_dt = 0.1
time (sec)
0
0.5
1
1.5
2
2.5
3
3.5



3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
0.5
1
1.5
2
2.5
3
time (sec)
Unstable Operation
M = 5, A = 2,
tau_dt = 0.2
signal C: solid
signal B: dotted
signal 0.1*M: dashed