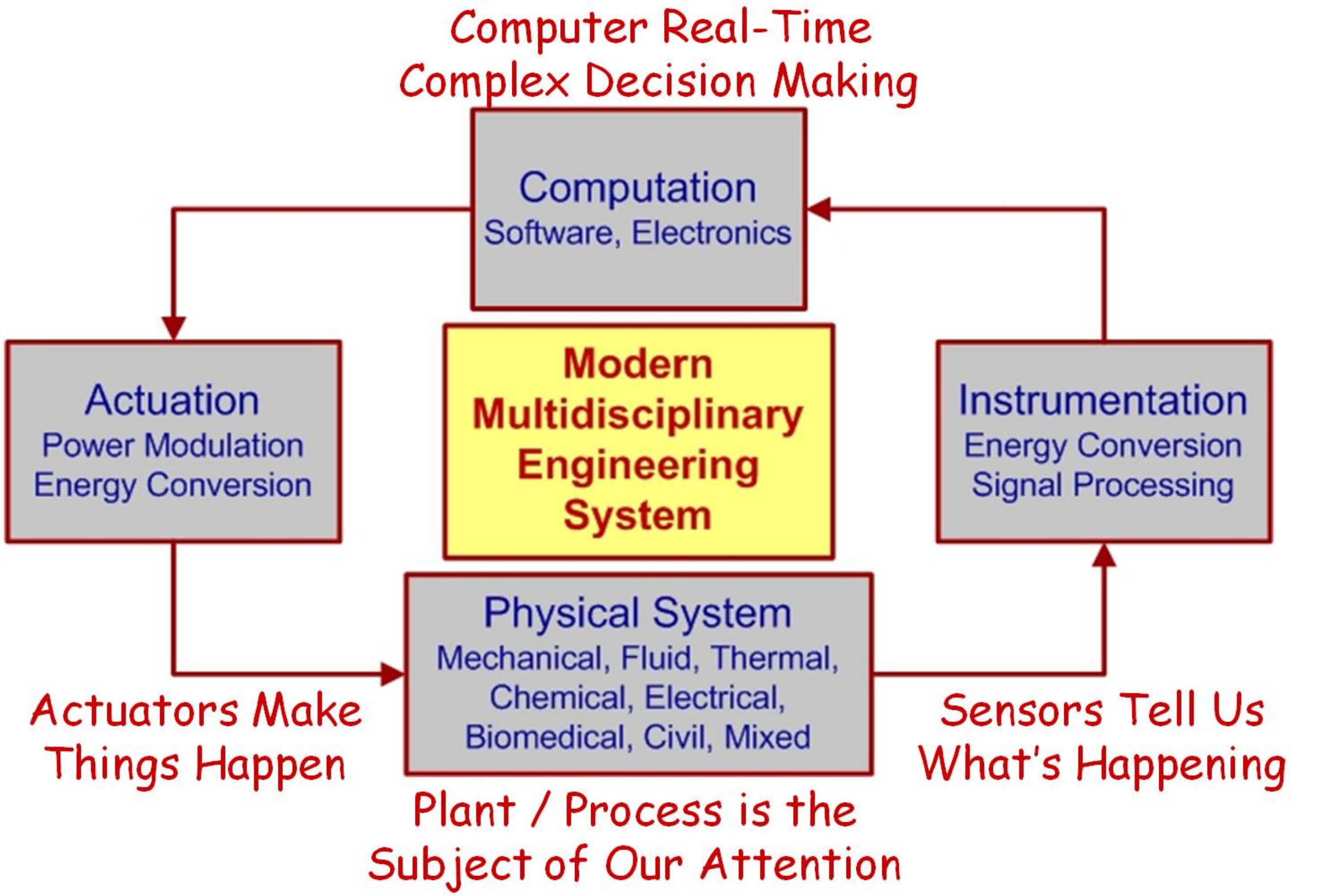
Electrical Systems:
Modeling, Analysis, Measurement, & Control



Electrical System Topics

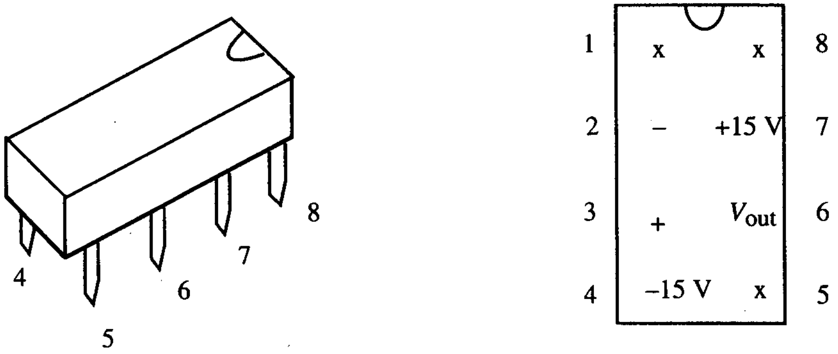
- Part 4
  - Op-Amps
  - Comparator and Schmitt Trigger

The Operational Amplifier

- Op-Amps are possibly the most versatile linear integrated circuits used in analog electronics.
- The Op-Amp is not strictly an element; it contains elements, such as resistors and transistors. However, it is a basic building block, just like R, L, and C.
- We treat this complex circuit as a black box!
  - Do we know all about the internal details? No!
  - Do we know how to use it and interface it with other electronic components? Yes, we must!

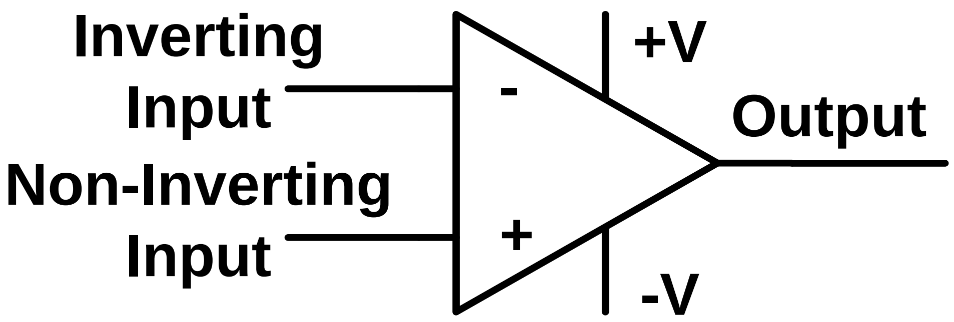
- Coincidently, the op-amp is a small black box with 8 connectors (only 5 are usually used).
- Op-Amps – Operational Amplifiers – are so called because they can be used to perform mathematical operations on input signals: addition, subtraction, multiplication, division, differentiation, and integration.
- Other common uses include:
  - Impedance buffering
  - Active filters
  - Active controllers
  - Analog-digital interfacing

741 Op Amp



- The op-amp has two inputs, an inverting input (-) and a non-inverting input (+), and one output. The output goes positive when the non-inverting input (+) goes more positive than the inverting (-) input, and vice versa. The symbols + and – do not mean that that you have to keep one positive with respect to the other; they tell you the relative phase of the output.

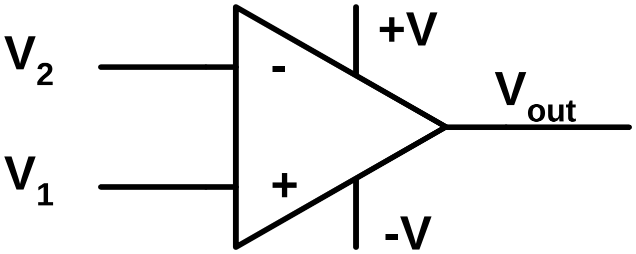
A fraction of a millivolt between the input terminals will swing the output over its full range.



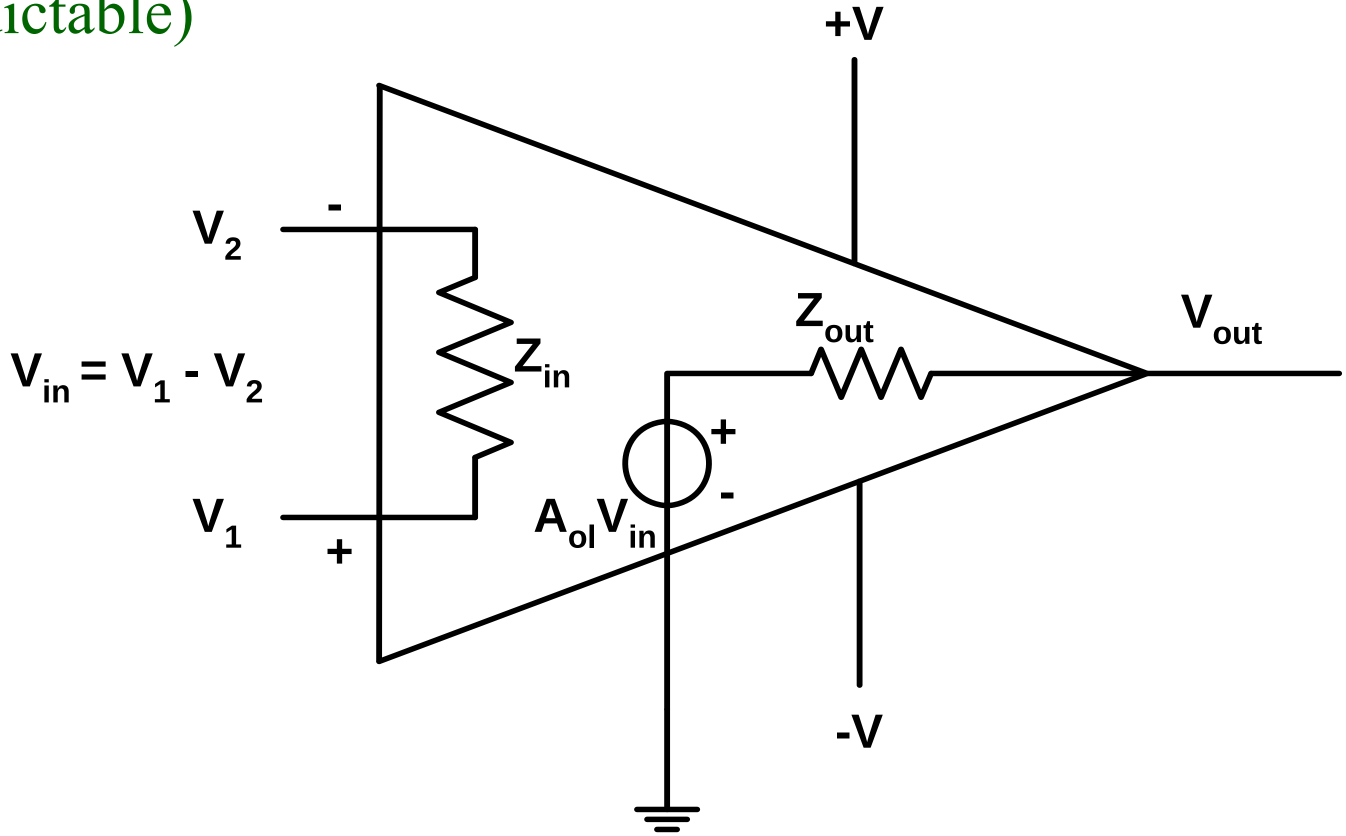
- Formal Definition of an Op-Amp:
  - dc-coupled: the op amp can be used with ac and dc input voltages
  - differential voltage amplifier: the op amp has two inputs
  - single-ended low-resistance output: the op amp has one output whose voltage is measured with respect to ground.
  - very high input resistance
  - very high voltage gain: op amp is a good voltage amplifier

$$\frac{V_{out}}{V_{in}} = A_{open\ loop} \approx 10^5 - 10^6$$

$$V_{in} = V_1 - V_2$$



- Real Op Amp
  - Z_{in} is the input impedance (very large ≈ 2 MΩ)
  - Z_{out} is the output impedance (very small ≈ 75 Ω)
  - A_{ol} is the open-loop gain (very large ≈ 200,000 and unpredictable)



- The op-amp input looks like a load circuit to any circuit connected to its input.
- The circuit driving the op amp "sees" the input impedance Z_{in}.
- The op amp is a voltage source and we represent it by its Thevenin equivalent circuit. An ideal op amp looks like an ideal voltage source; a real op amp has a real voltage source as an output.

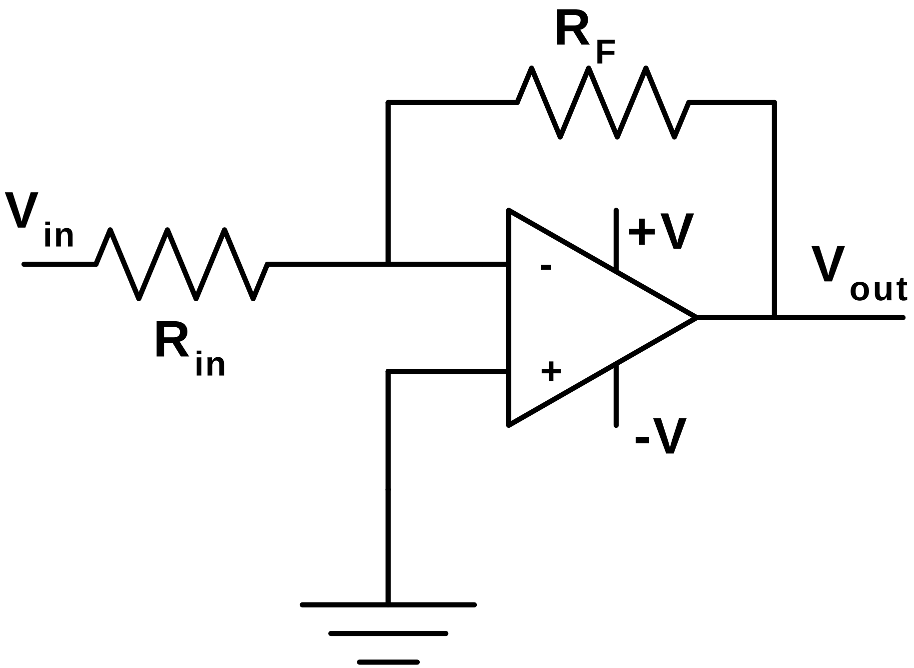
- Since operational amplifiers have enormous and unpredictable voltage gain (10^6 or so), they are never used without negative feedback.
- Negative feedback is the process of coupling the output back in such a way as to cancel some of the input. This does lower the amplifier's gain, but in exchange it also improves other op-amp characteristics, such as:
  - Freedom from distortion and nonlinearity
  - Flatness of frequency response or conformity to some desired frequency response
  - Stability and Predictability
  - Insensitivity to variation in A_{ol}

- As more negative feedback is used, the resultant amplifier characteristics become less dependent on the characteristics of the open-loop (no feedback) amplifier and finally depend only on the properties of the feedback network itself. For example:

Basic Inverting Op-Amp

$$\text{Gain} = \frac{R_F}{R_{in}}$$

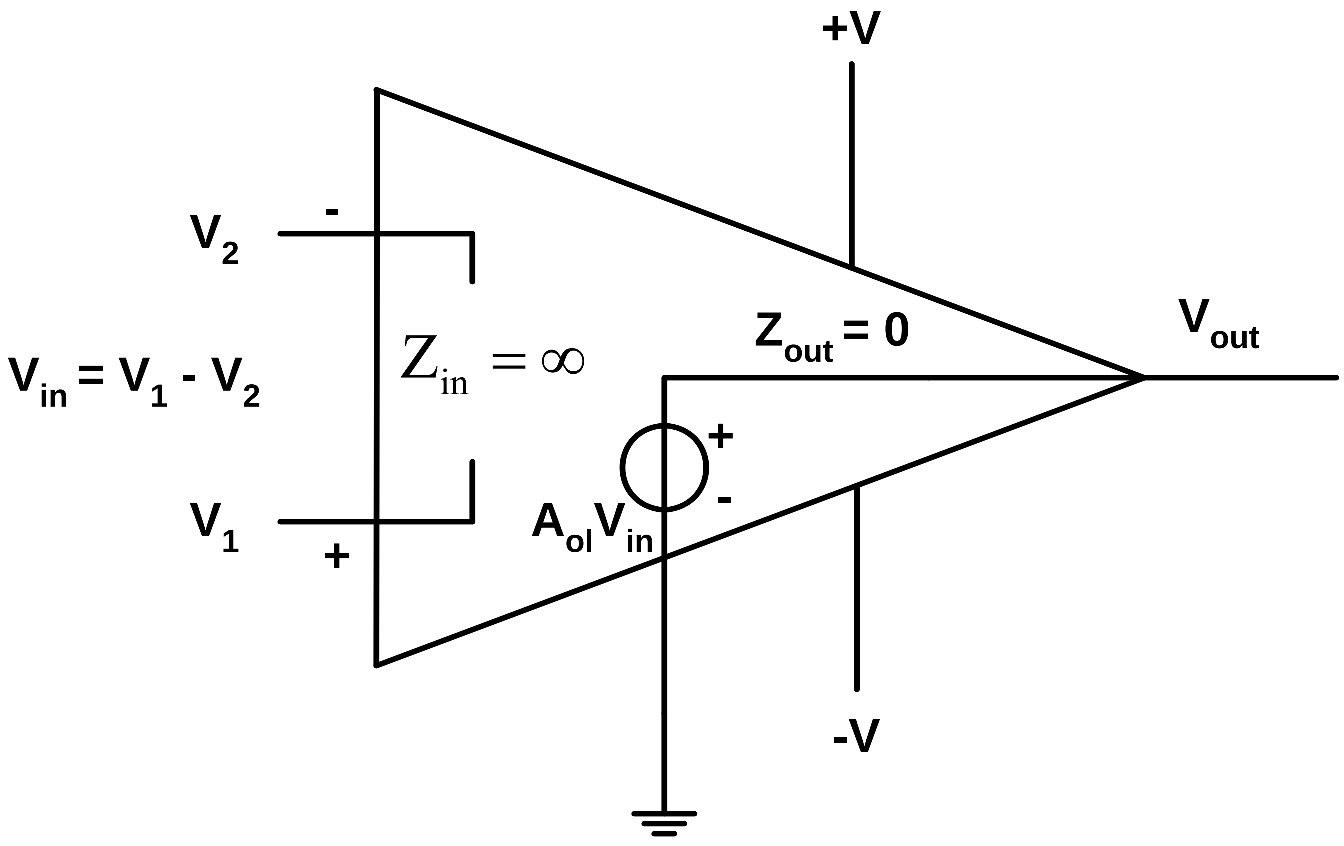
$$V_{out} = -V_{in}\frac{R_F}{R_{in}}$$



- A properly designed op-amp allows us to use certain simplifying assumptions when analyzing a circuit which uses op-amps; we accept these assumptions "on faith." They make op-amp circuit analysis quite simple.
- The so-called "golden rules" for op-amps with negative feedback are:
  – The output attempts to do whatever is necessary to make the voltage difference between the inputs zero. The op-amp "looks" at its input terminals and swings its output terminal around so that the external feedback network brings the input differential to zero.
  – The inputs draw no current (actually < 1 nA).

- Simplifying Assumptions:
  - The op-amp's gain A_{ol} is infinite
  - Z_{in} is infinite; thus no current is drawn at the input terminals
  - Z_{out} is zero; thus $V_{out} = A_{ol}V_{in}$
  - The time response is instantaneous
  - The output voltage has a definite design range, such as ± 15 volts, since it can never be greater than +V or less than -V. Proper operation is possible only for output voltages within these limits. If saturation occurs, the golden rules may no longer be true.

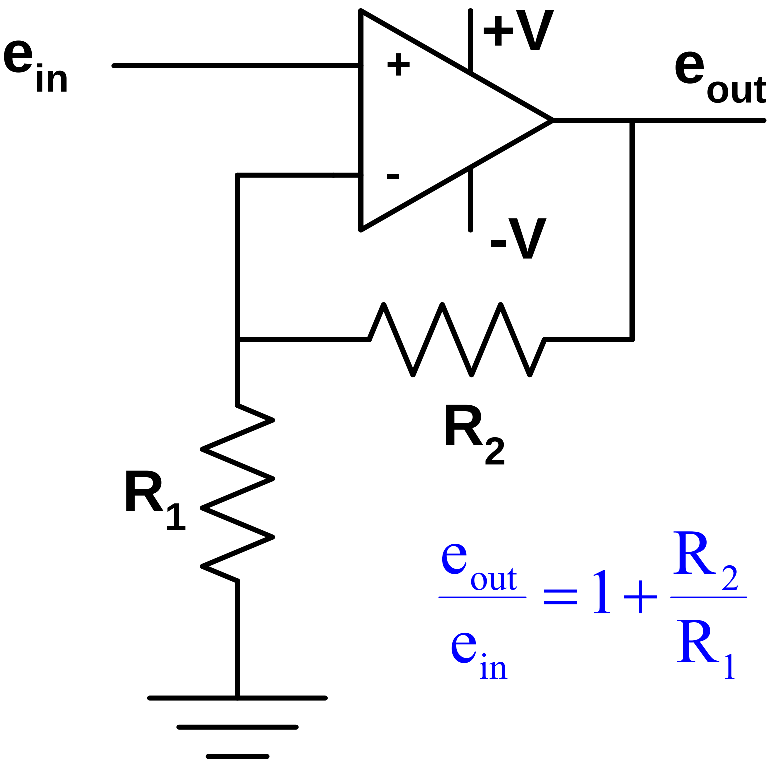
- Ideal Op Amp based on the simplifying assumptions



- <u>Basic Op-Amp Cautions</u>
  - In all op-amp circuits, the “golden rules” will be obeyed only if the op-amp is in the active region, i.e., inputs and outputs are not saturated at one of the supply voltages. Note that the op-amp output cannot swing beyond the supply voltages. Typically it can swing only to within 2V of the supplies.
  - The feedback must be arranged so that it is negative; you must not mix the inverting and non-inverting inputs.
  - There must always be feedback at DC in the op-amp circuit. Otherwise, the op-amp is guaranteed to go into saturation.

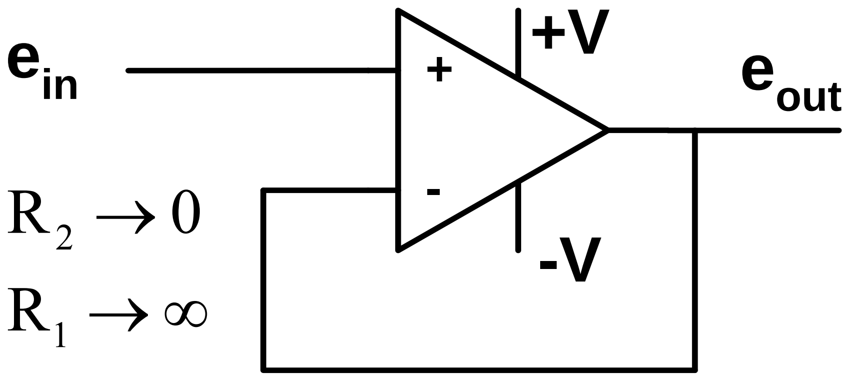
- Many op-amps have a relatively small maximum differential input voltage limit. The maximum voltage difference between the inverting and non-inverting inputs might be limited to as little as 5 volts in either polarity. Breaking this rule will cause large currents to flow, with degradation and destruction of the op-amp.
- Note that even though op-amps themselves have a high input impedance and a low output impedance, the input and output impedances of the op-amp circuits you will design are not the same as that of the op-amp.

- <u>Non-Inverting and Unity-Gain Buffer Op-Amps</u>



$$\frac{e_{out}}{e_{in}} = 1 + \frac{R_2}{R_1}$$

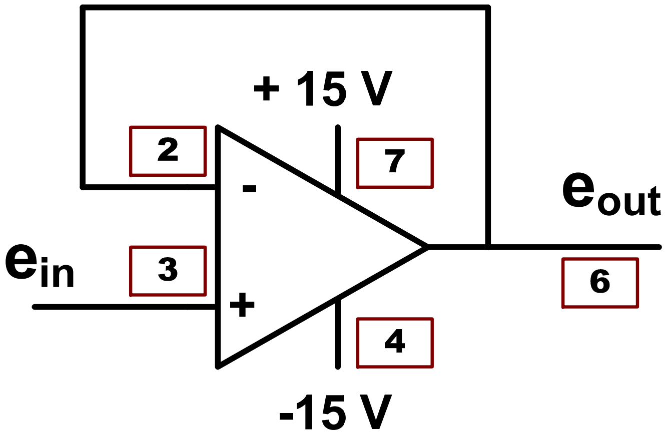
Non-Inverting Op-Amp



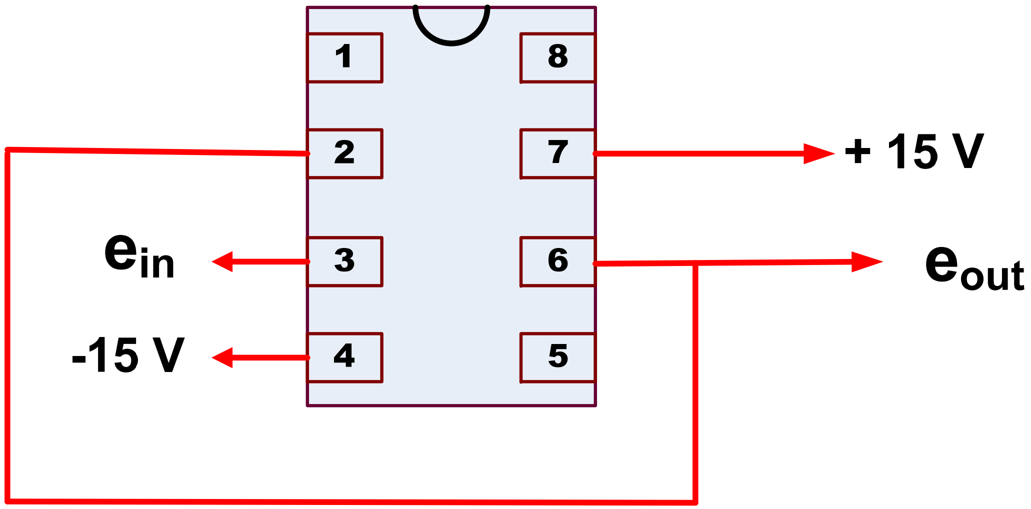
$$R_2 \to 0$$
$$R_1 \to \infty$$

$$\frac{e_{out}}{e_{in}} = 1$$

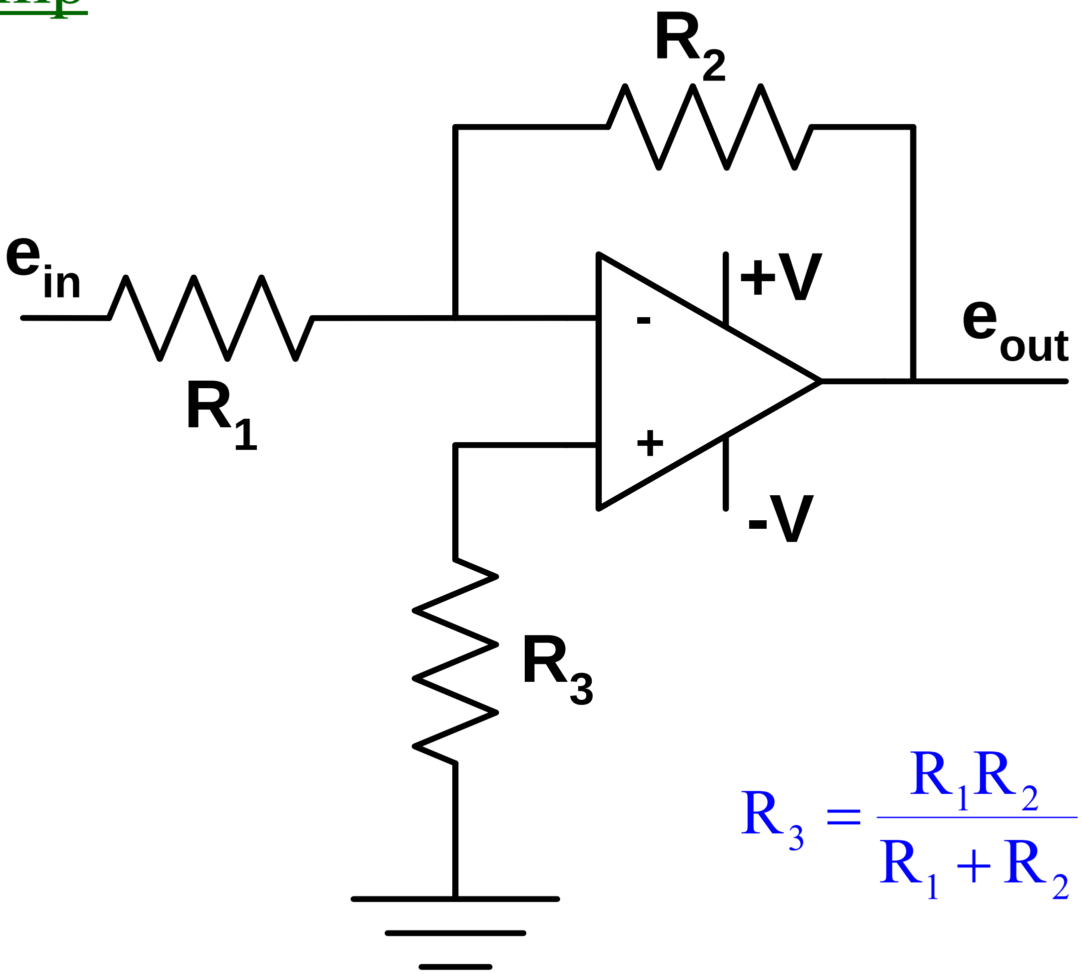
Unity-Gain, Buffer Op-Amp



Unity Gain Buffer Op Amp



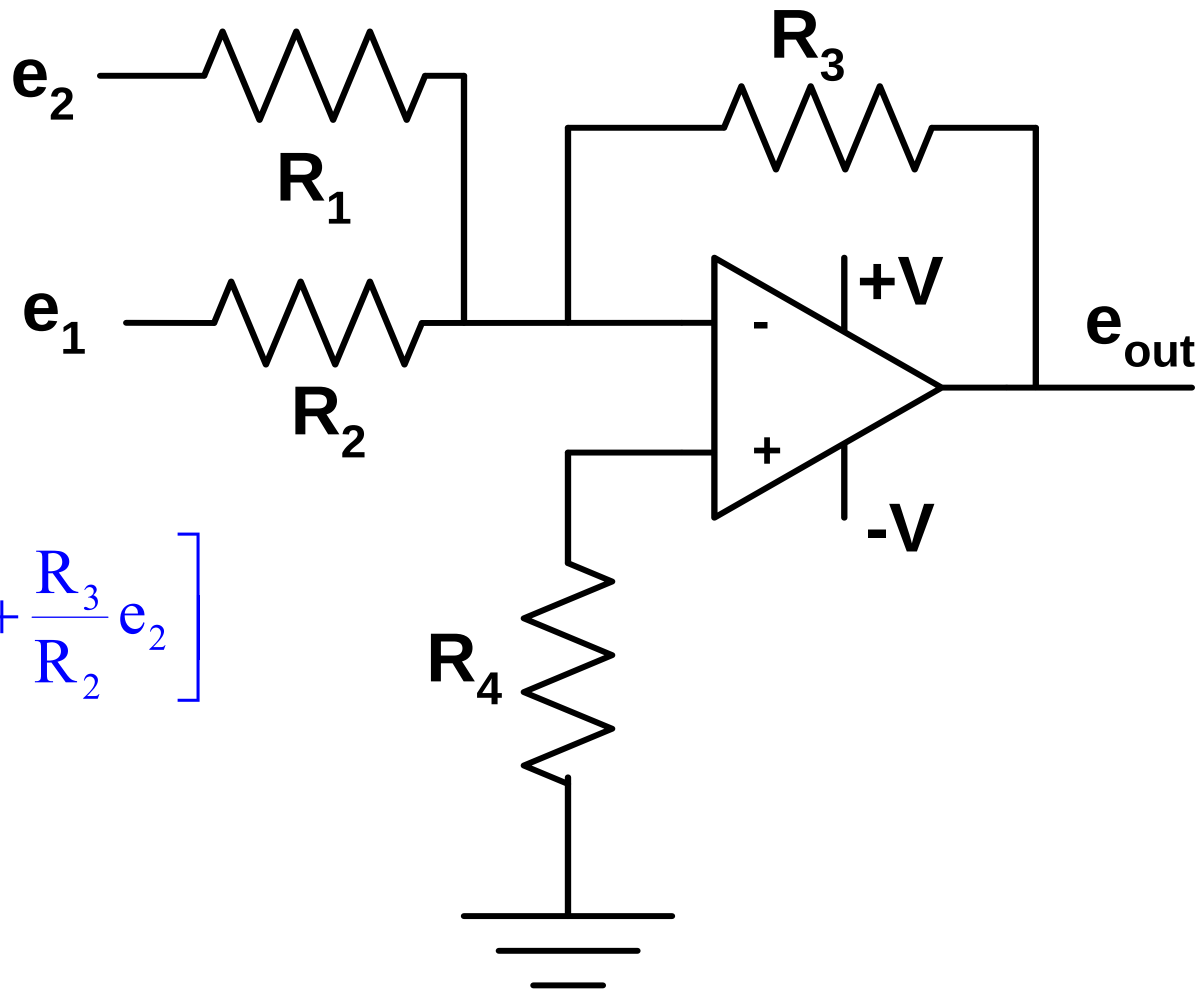
- Inverting Op-Amp



$$\frac{e_{out}}{e_{in}} = -\frac{R_2}{R_1}$$

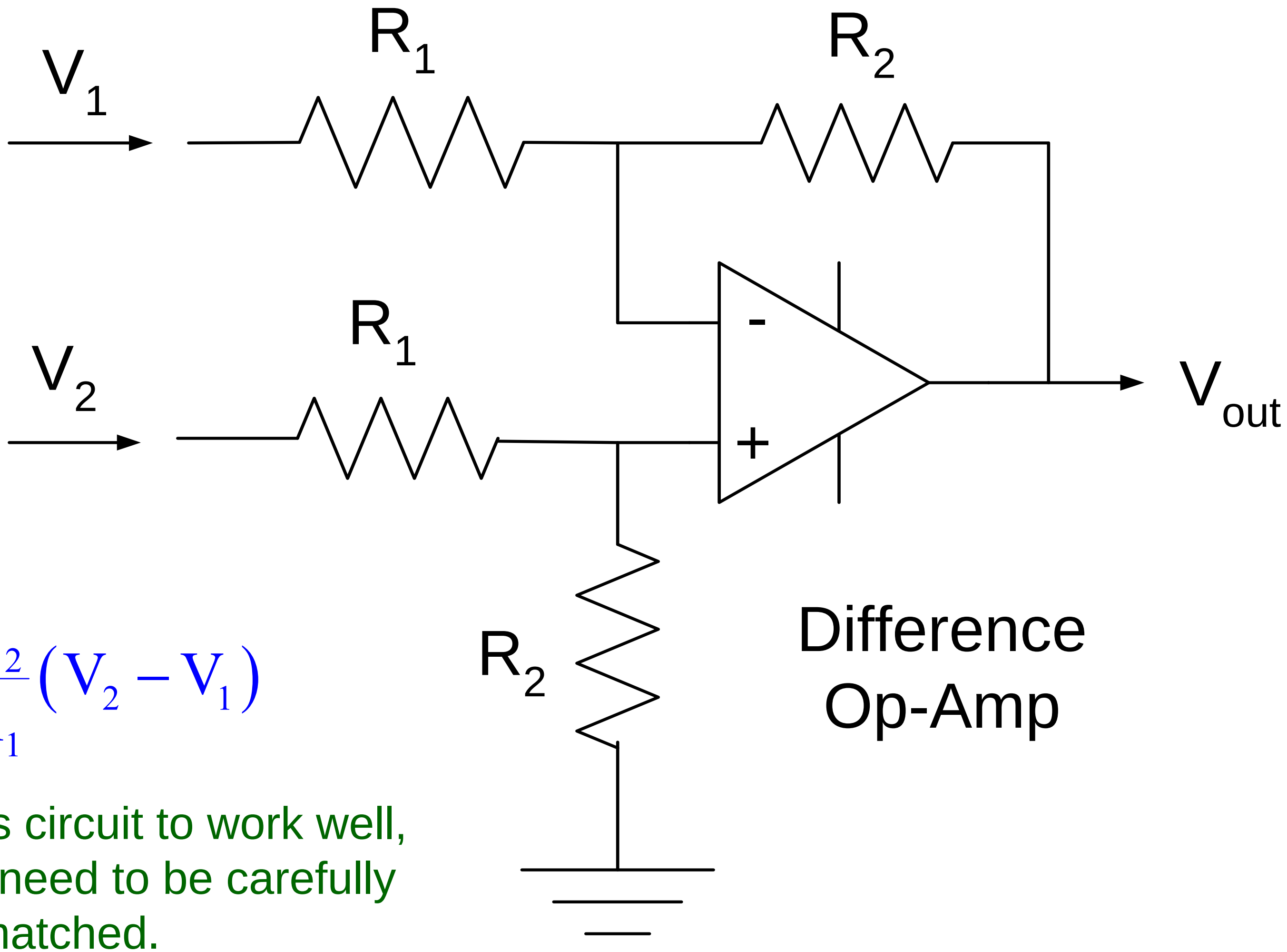
$$R_3 = \frac{R_1 R_2}{R_1 + R_2}$$

- ## Summing Op-Amp



$$e_{out} = -\left[\frac{R_3}{R_1}e_1 + \frac{R_3}{R_2}e_2\right]$$

- <u>Difference Op-Amp</u>

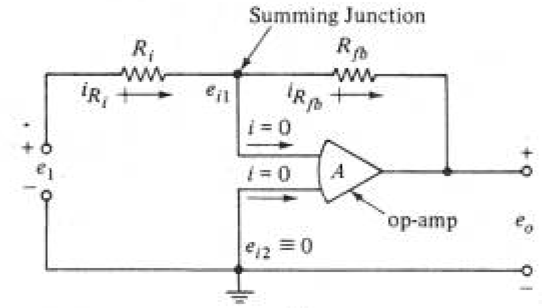


$$V_{out} = \frac{R_2}{R_1}\left(V_2 - V_1\right)$$

Note: For this circuit to work well, the resistors need to be carefully matched.

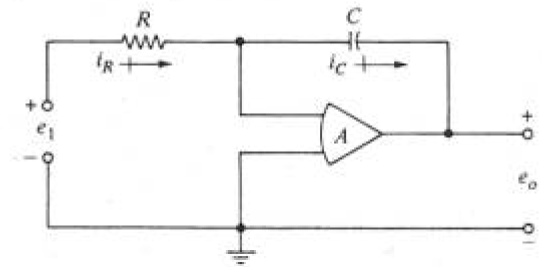
Rarely used by itself, the op-amp is usually combined with passive elements, mainly resistors and capacitors.

Coefficient Multiplier or Inverter



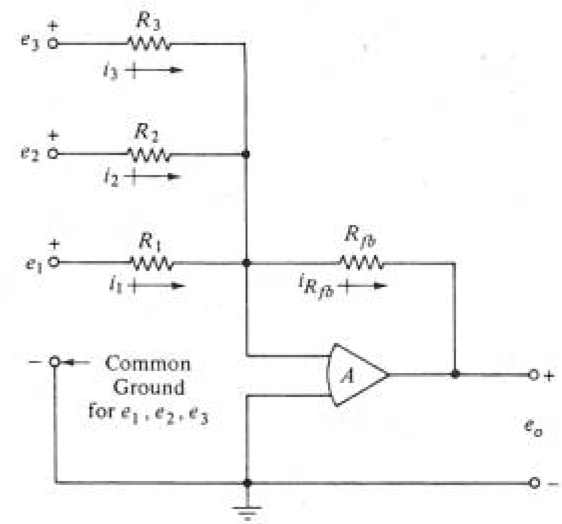
(a)

Integrator



(b)

Summer



(c)

- To analyze op-amp circuits we use two basic electrical circuit laws, Kirchhoff's voltage loop and current node laws, together with the op-amp simplifying assumptions.
- Let's analyze the coefficient multiplier circuit:

$$i_{R_i} = i_{R_{fb}}$$

$$\frac{e_1 - e_{i1}}{R_i} = \frac{e_{i1} - e_o}{R_{fb}}$$

$$e_o = -Ae_{i1}$$

$$\frac{e_1 + \frac{e_o}{A}}{R_i} = \frac{\frac{-e_o}{A} - e_o}{R_{fb}}$$

$$e_o = -\frac{R_{fb}}{R_i} e_1 \qquad \text{if } A = \infty$$

- Of course A cannot be infinite, but it can be, say, 10^6 volts/volt, and then this is a good approximation.

- Why not use the op-amp directly as an amplifier since it has more than enough gain?
  - The op-amp gain can be relied upon to be very large but cannot be relied upon to be an accurate stable value.
  - The gain A is guaranteed to be, say, in the range 1 to 5 million V/V.
  - As long as A is large enough, our approximation is valid.
  - Also note that in the multiplier example, the accuracy and stability depends on the values of the two resistors and not on the value of A, as long as A is large enough.
  - Using op-amps, we can construct circuits whose performance depends mainly on passive components selected to have accurate and stable values.

- *A* is called the *open-loop gain*, while e_o/e_1 in the example is called the *closed-loop gain*, which in this case due to the circuit configuration, is negative.
- Since *A* may be treated as infinite, the voltage e_{i1}, the summing junction voltage, can always be treated as zero in those op-amps where the positive input is grounded. The summing junction is known as *virtual ground*, since its voltage is for all practical purposes zero, the same as true ground, whose voltage is exactly zero.
- When op-amps don't ground the positive input (differential input), the difference $(e_{i2} - e_{i1})$ is taken to be practically zero.
- Using these assumptions, we can analyze the integrator circuit as follows:

$$i_R = \frac{e_1 - 0}{R} = i_C = C\frac{d}{dt}\left(0 - e_o\right) = -CDe_o$$

$$e_o = -\frac{1}{RCD}e_1 = -\frac{1}{RC}\int e_1 dt$$

- Similarly, for the summer circuit:

$$i_1 + i_2 + i_3 = i_{R_{fb}}$$

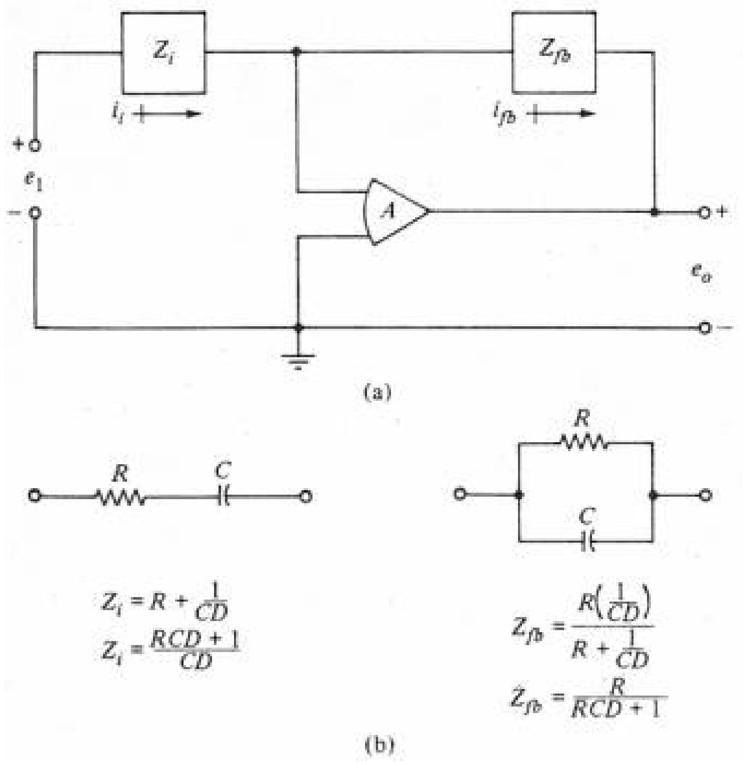
$$\frac{e_1 - 0}{R_1} + \frac{e_2 - 0}{R_2} + \frac{e_3 - 0}{R_3} = \frac{0 - e_o}{R_{fb}}$$

$$e_o = -\left(\frac{R_{fb}}{R_1} e_1 + \frac{R_{fb}}{R_2} e_2 + \frac{R_{fb}}{R_3} e_3 \right)$$

$$e_o = -\left(e_1 + e_2 + e_3 \right) \quad \text{if } R_{fb} = R_1 = R_2 = R_3$$

- While multiplier, integrator, and summer are fundamental operations for solving differential equations, op-amps have many other uses.

- Examples are high-pass filters, low-pass filters, band-pass filters, band-reject filters, lead controllers, lag controllers, lead-lag controllers, approximate integrators and differentiators.



Z_i and Z_o represent arbitrary impedances

$$i_1 = \frac{e_1 - 0}{Z_i(D)} = i_{fb} = \frac{0 - e_o}{Z_{fb}(D)}$$

$$\frac{e_o}{e_1}(D) = -\frac{Z_{fb}(D)}{Z_i(D)}$$

$$\frac{e_o}{e_1}(D) = \frac{-R_2C_1D}{(R_1C_1D+1)(R_2C_2D+1)}$$

- Deviations of Real Op-Amps from Ideal Assumptions
  - Effect of non-infinite gain A
    - For the multiplier circuit

$$\frac{e_1 + \frac{e_o}{A}}{R_i} = \frac{\frac{-e_o}{A} - e_o}{R_{fb}}$$

$$e_o = -\frac{R_{fb}}{R_i} e_1 \left(1 + \frac{1}{A} + \frac{R_{fb}}{AR_i} \right)$$

    - The open-loop gain A may be in the range of 10^4 to 10^8, while R_{fb}/R_i rarely exceeds 10^3; thus the error upper limit is from about 10^{-5} to 10^{-1}. If one selects precision resistors so as to get a precise e_o/e_1, if the gain A is too low, the ratio will be inaccurate.

- Offset voltage
  - Offset voltage refers to the fact that if e_1 is made zero by grounding it, e_o will not be exactly zero, due to imperfections in the amplifier.
  - The best values of offset voltage e_{os} are the order of 30 μV over a temperature range 0f –25 to +85°C, with a temperature coefficient of about 0.2 μV/ °C.
  - Op-amps can be trimmed using some additional circuitry with adjustable resistors to eliminate this offset.
- Bias Current
  - Bias current is the small current that flows in the amplifier input leads, even when no input voltage is applied.
  - Values of i_{b1} can be as small as 75E-15 amps at 25 °C, and would never exceed ± 4 pA.

– Non-infinite Input Impedance and Nonzero Output Impedance

- Analysis shows that the effect of non-infinite input impedance is equivalent to a loss of open-loop gain A, the effective value being given by:

$$A_{eff} = \frac{A}{1 + \dfrac{R_i R_{fb}}{R_1 \left(R_i + R_{fb} \right)}}$$

- A similar effect is produced by nonzero output impedance (R_L is a load resistance representing the input resistance of any device which would be connected to the op-amp circuit):

$$A_{eff} = \frac{A}{1 + \dfrac{R_2}{R_{fb}} + \dfrac{R_2}{R_L}}$$

- Input resistances are in the range of 10^6 to 10^{13} ohms while output resistances are the order of 100 ohms.

– <u>Speed of Response</u>

- The speed of response is specified in several different ways. One method considers the closed-loop frequency response when the op-amp is connected as coefficient multiplier. The fastest op-amps will have this frequency response flat to about 500 MHz when the input resistance and feedback resistance are set equal, i.e., a closed-loop gain of 1. For a closed-loop gain of 20, the flat range of amplitude drops to about 80 MHz.

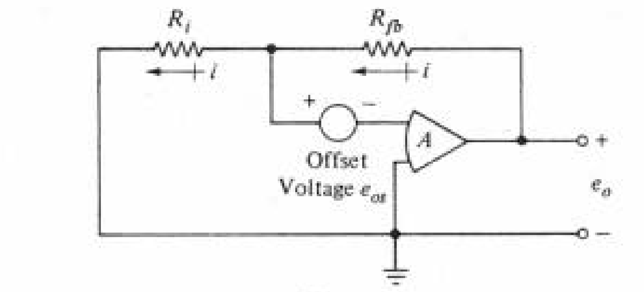
- Another method uses settling time after a step input is applied. Times to settle within 1, 0.1, and 0.01% of the final value may be quoted. The settling time is typically a few nanoseconds.

– Power Limitations

- Most op-amps supply only limited electrical power at their output terminals, e.g., ± 10 volts and 0.05 amps.

Offset Voltage

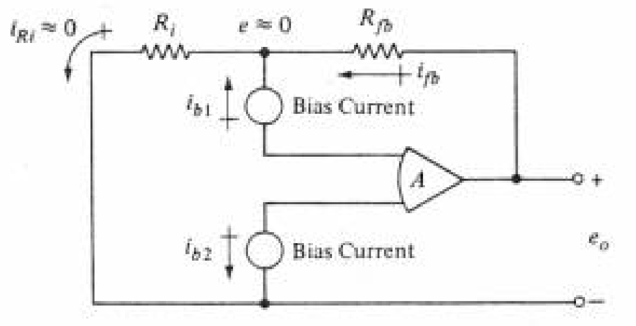
$$e_o = e_{os}\left(1+\frac{R_{fb}}{R_i}\right)$$



(a)

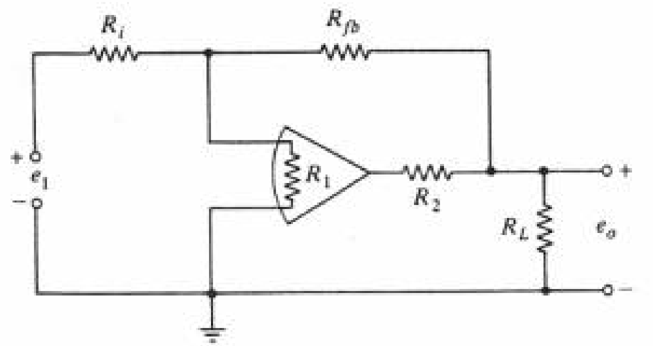
Bias Current

$$e_o = -i_{b1}R_{fb}$$



(b)

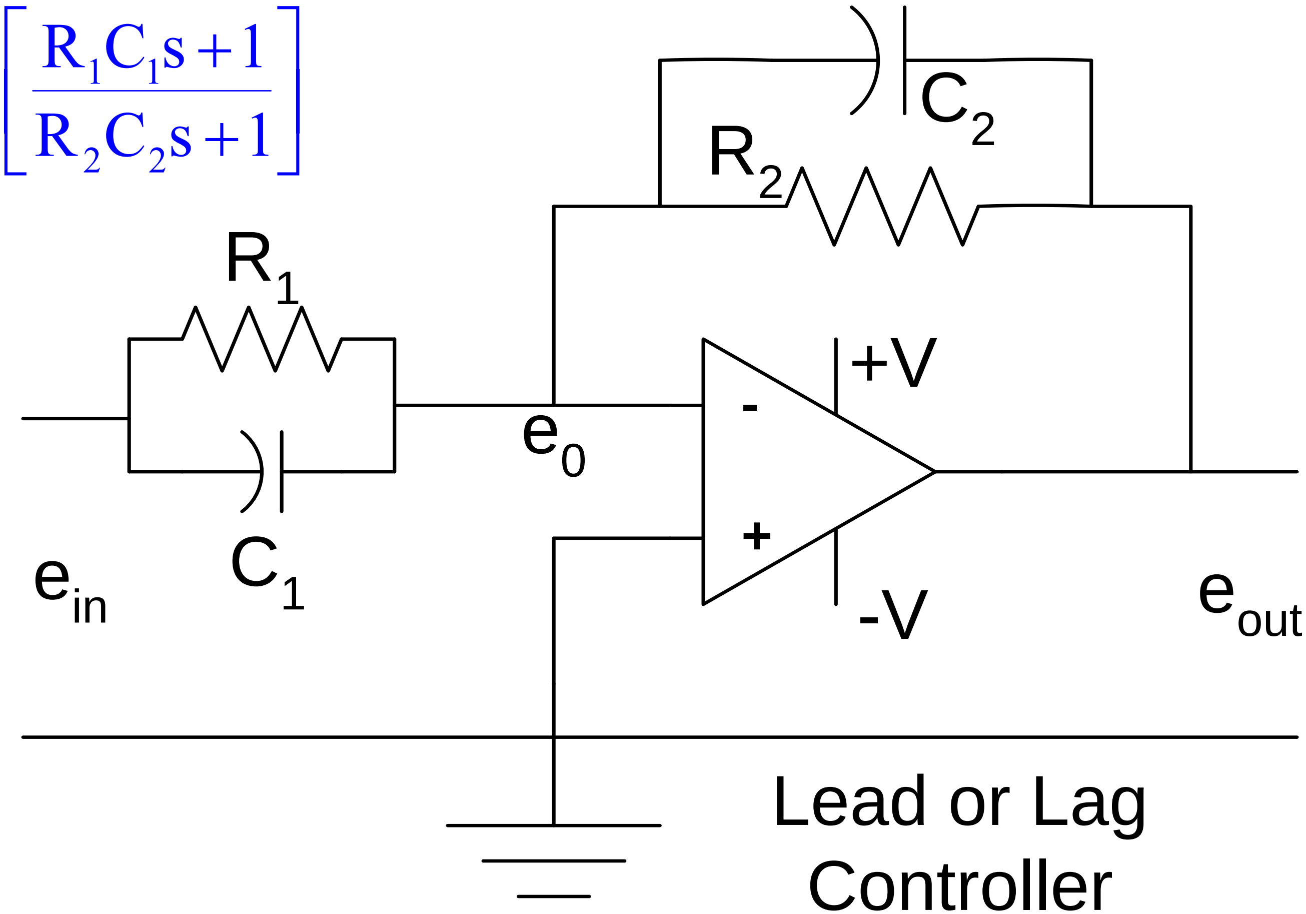
Non-infinite Input Impedance and Nonzero Output Impedance



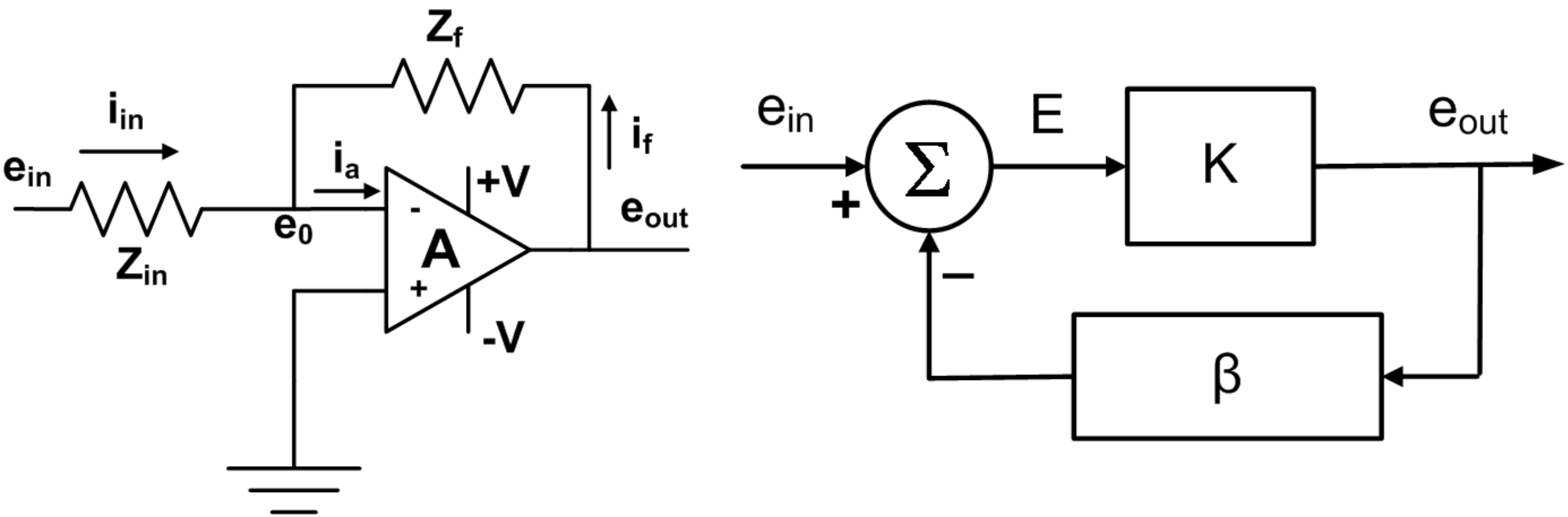
(c)

Active Lead / Lag Controller

$$\frac{e_{out}}{e_{in}} = \left[-\frac{R_2}{R_1} \right] \left[\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} \right]$$



- Passive Control
  - All sensing and actuation functions are integrated within the subsystem, and independent energy sources are usually not needed. There are no distinct components for sensing and actuation.
  - The Gain-Setting Circuit for an Inverting Amplifier is a classic example of a passive feedback controller.



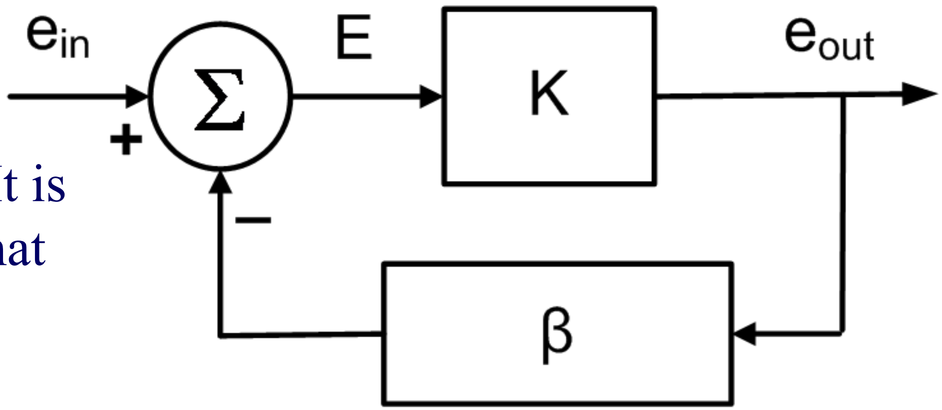
- Background on Non-Ideal Op-Amp Equations
  - There are two sources of error in op amps: DC errors and AC errors.
    - Examples of DC errors are input offset voltage and input bias current. These errors stay constant over the usable op-amp frequency range.
    - AC errors show up under DC conditions, but they get worse as the operating frequency increases. Differential gain (op-amp gain or op-amp open-loop gain) is the most important AC specification as other AC specifications (e.g., output impedance, peak-to-peak output voltage, power-supply rejection ratio) are derived from it.

- Amplifiers are built with active components such as transistors, whose parameters are subject to drift and inaccuracies. The drift and inaccuracy in circuits using these components are minimized or eliminated by using negative feedback to make the transfer function of the circuit independent of the amplifier parameters and dependent on external passive components.
- A block diagram for a generalized feedback system is shown.

$$\frac{e_{out}}{e_{in}} = \frac{K}{1+\beta K}$$

$$\frac{e_{out}}{e_{in}} = \frac{1}{\beta} \quad \beta K >> 1$$

βK is called the Loop Gain. It is the sole factor that determines stability.



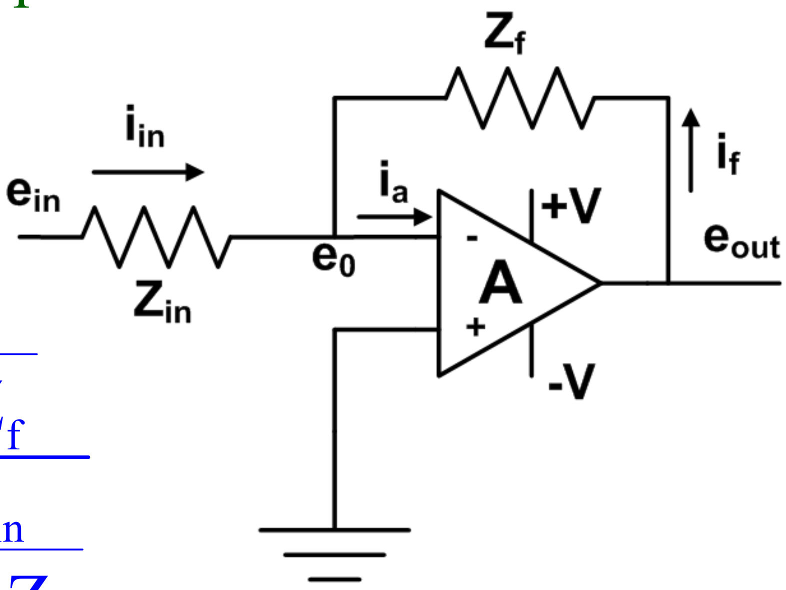
– Error E is proportional to the input signal and inversely proportional to the loop gain. As the loop gain increases, the error decreases. Large loop gains are attractive for minimizing errors, but they also decrease stability. There is always a tradeoff between error and stability.

$$E = \frac{e_{in}}{1+\beta K}$$

– Consider the Inverting Op Amp.

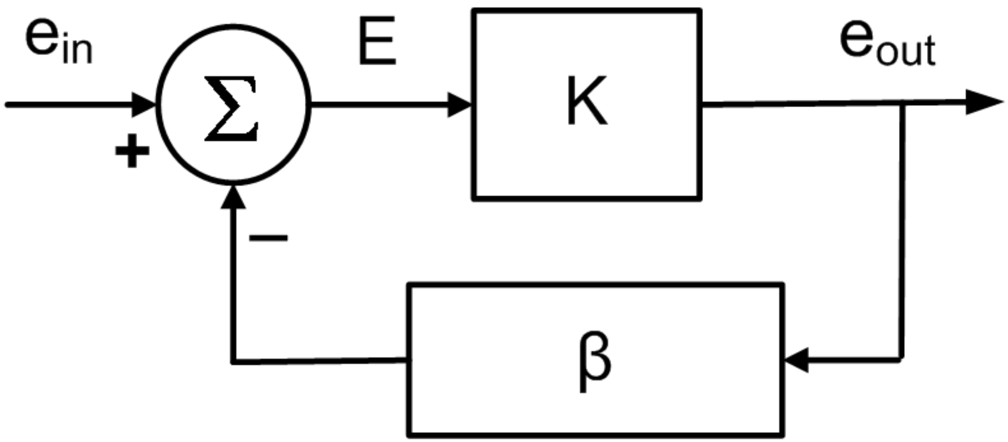
$$\left.\begin{aligned} e_{out} &= -e_o A \\ \frac{e_{in}-e_o}{Z_{in}} &= \frac{e_o - e_{out}}{Z_f} \end{aligned}\right\}$$

$$\frac{e_{out}}{e_{in}} = \frac{\dfrac{-AZ_f}{Z_{in}+Z_f}}{1+\dfrac{AZ_{in}}{Z_{in}+Z_f}}$$



– For the inverting op amp:

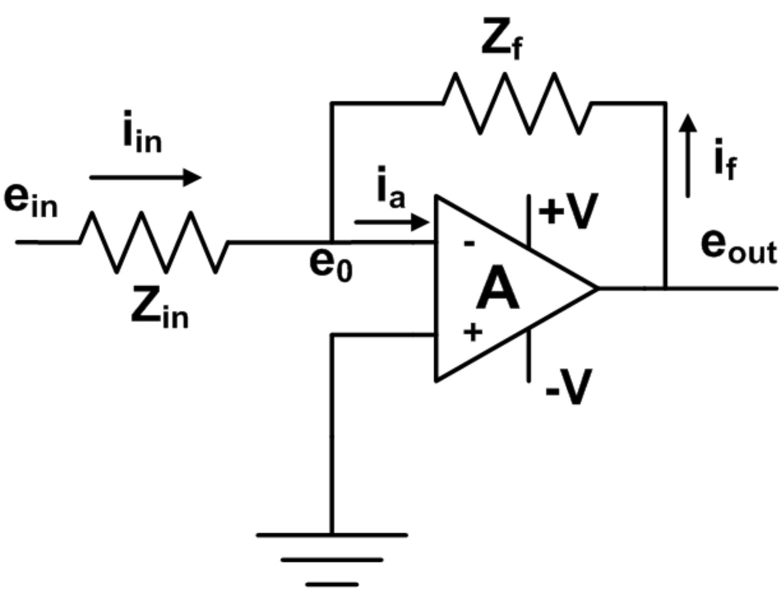
$$\frac{-e_{out}}{e_{in}} = \frac{K}{1+\beta K}$$



$$K = \frac{AZ_f}{Z_{in} + Z_f}$$

$$\beta = \frac{Z_{in}}{Z_f}$$

$$\beta K = \frac{AZ_{in}}{Z_{in} + Z_f}$$

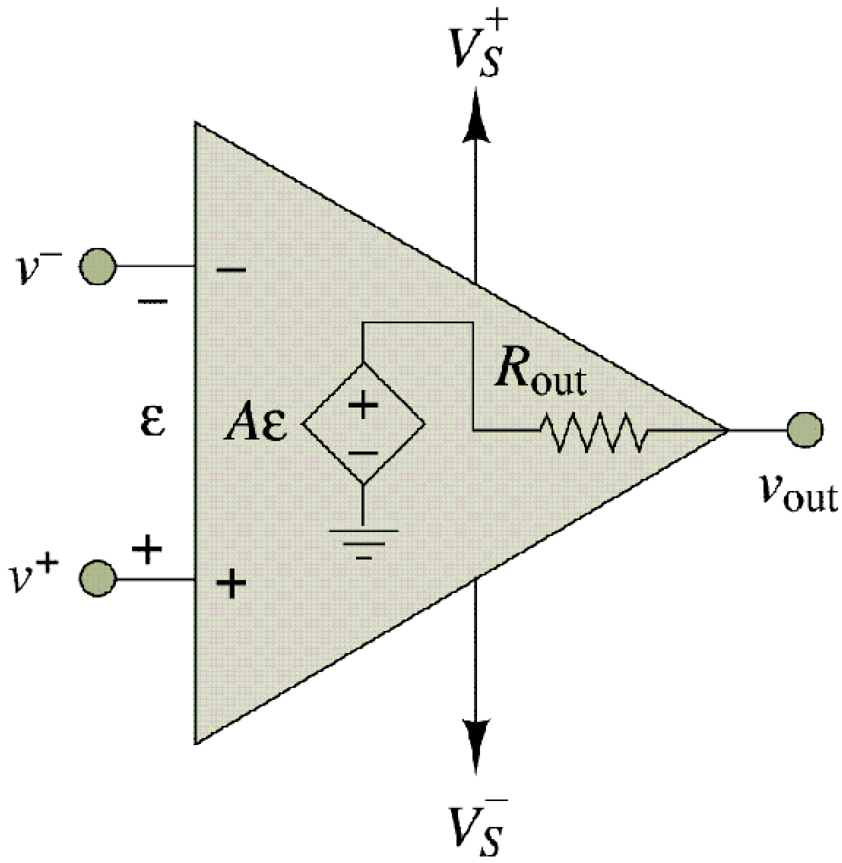


Comparator and Schmitt Trigger

- Comparator circuits find frequent application in measurement and instrumentation systems.
- Learning Objectives
  – Understand the Op-Amp Comparator with and without an offset and its uses
  – Understand the Schmitt Trigger and its benefits in noise rejection and improved switching speed

- <u>Op-Amp Comparator</u>
  - The prototype of op-amp switching circuits is the op-amp comparator.
  - The circuit does not employ feedback.

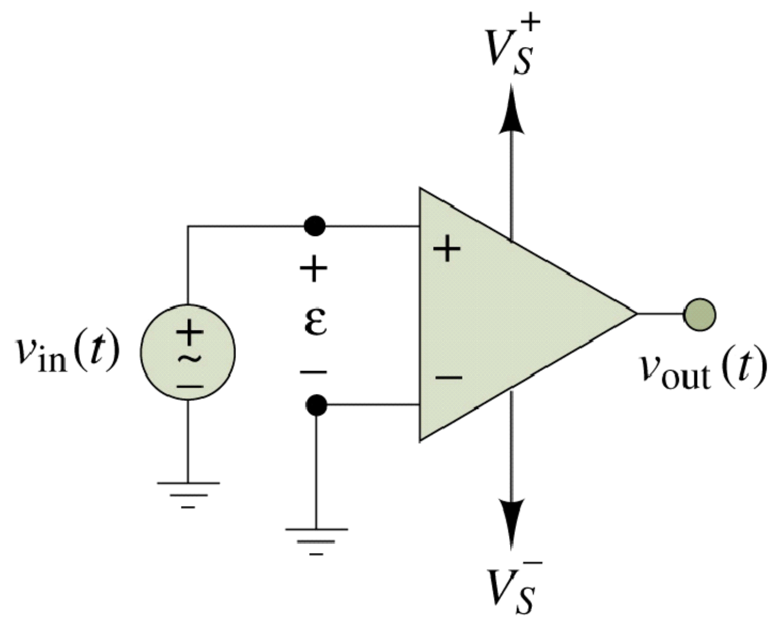
$$V_{out} = A\left(V^{+} - V^{-}\right)$$



- Because of the large gain that characterizes open-loop performance of the op-amp ($A > 10^5$), any small difference between the input voltages will cause large outputs; the op-amp will go into saturation at either extreme, according the voltage supply values and the polarity of the voltage difference.
- One can take advantage of this property to generate switching waveforms.
- Consider the following.

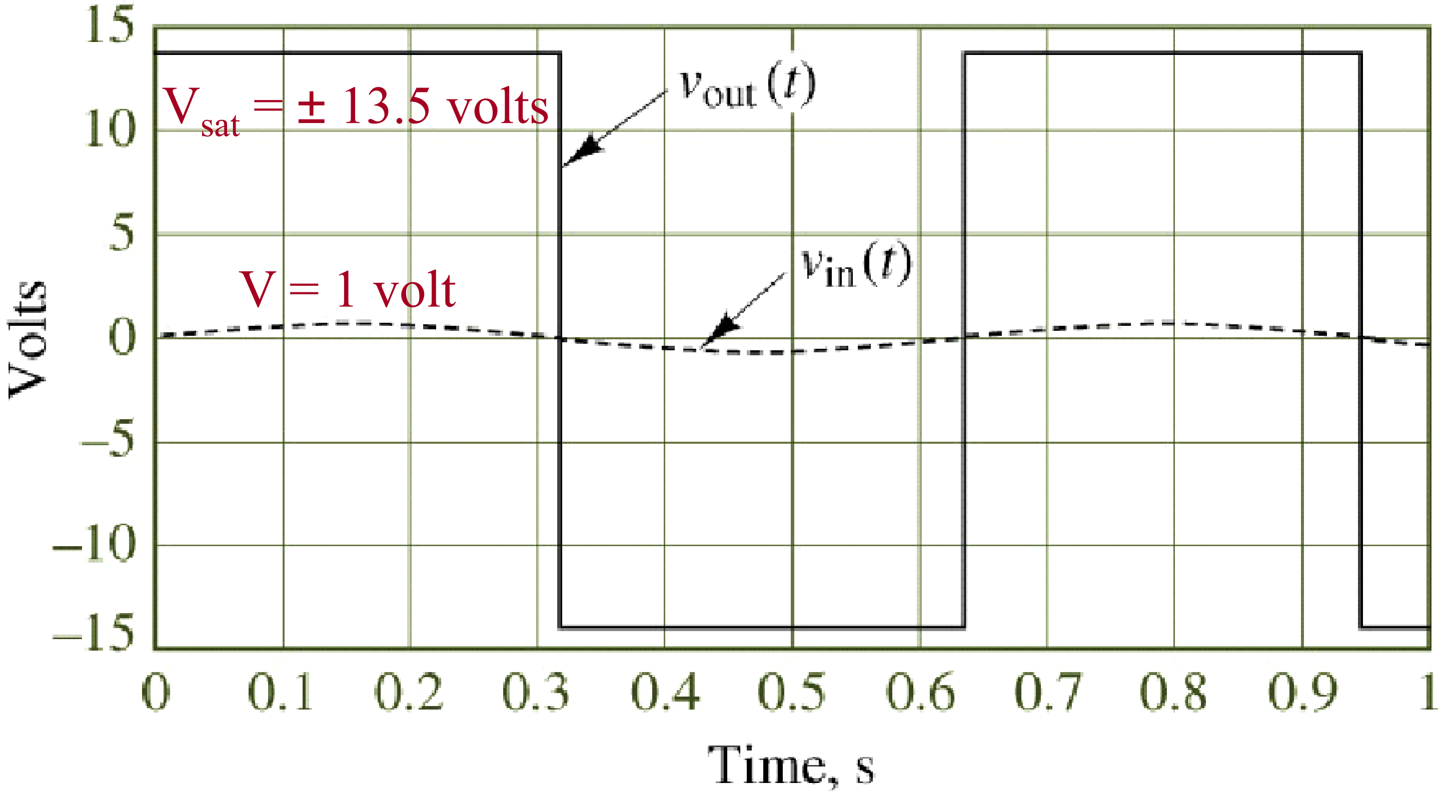
Non-inverting Op-Amp Comparator

$$\varepsilon = V\cos(\omega t)$$



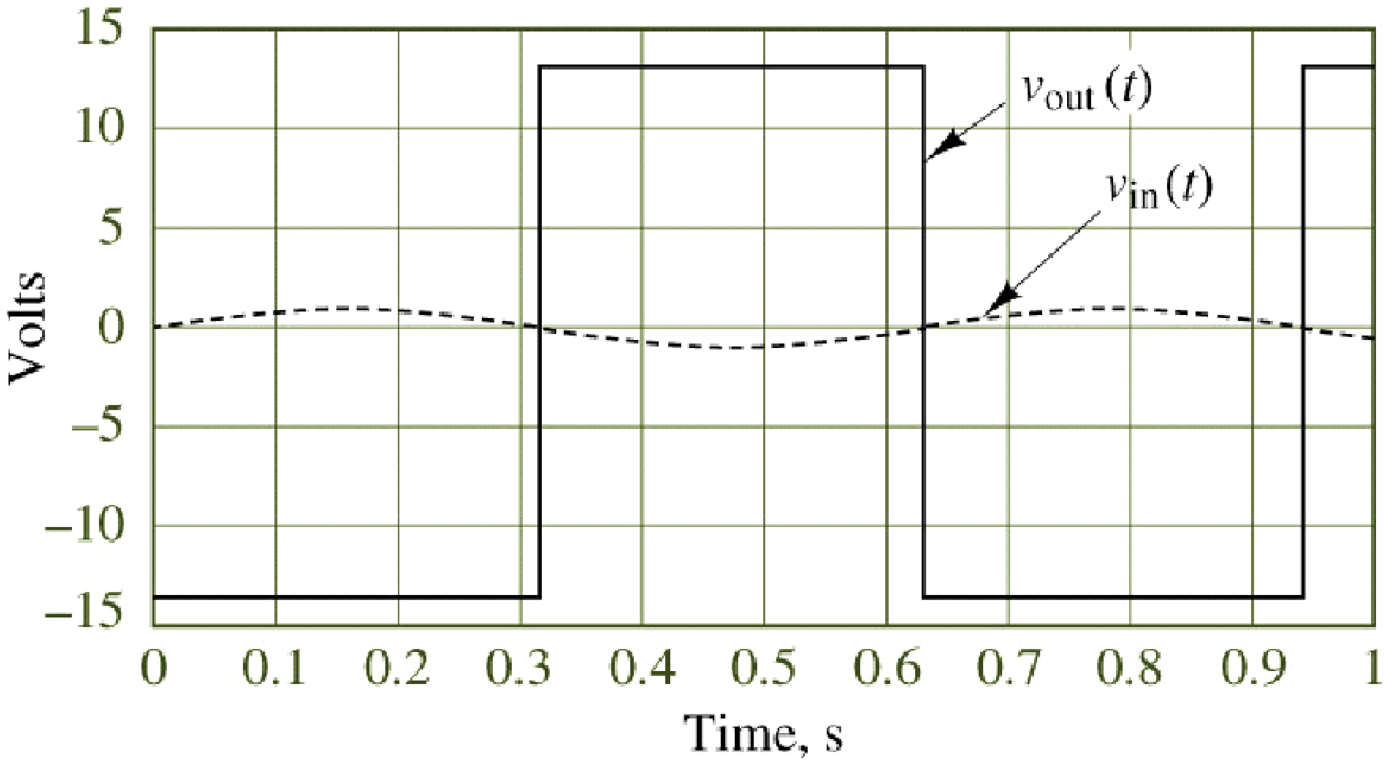
– The comparator is perhaps the simplest form on an analog-to-digital converter, i.e., a circuit that converts a continuous waveform to discrete values. The comparator output consists of only two discrete levels.

Input and Output of Non-Inverting Comparator



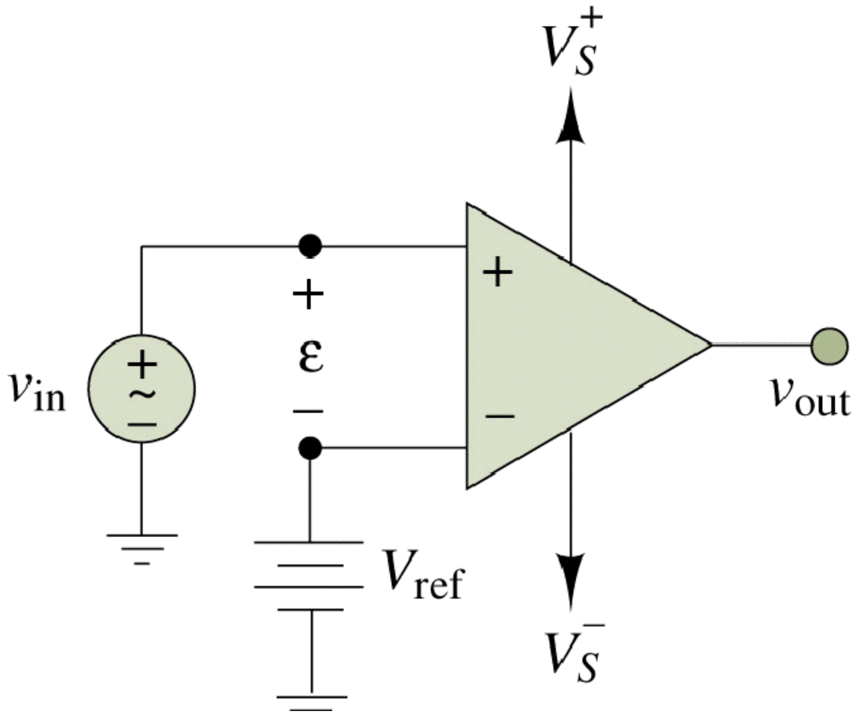
– It is possible to construct an inverting comparator by connecting the non-inverting terminal to ground and connecting the input to the inverting terminal.

Input and Output of Inverting Comparator

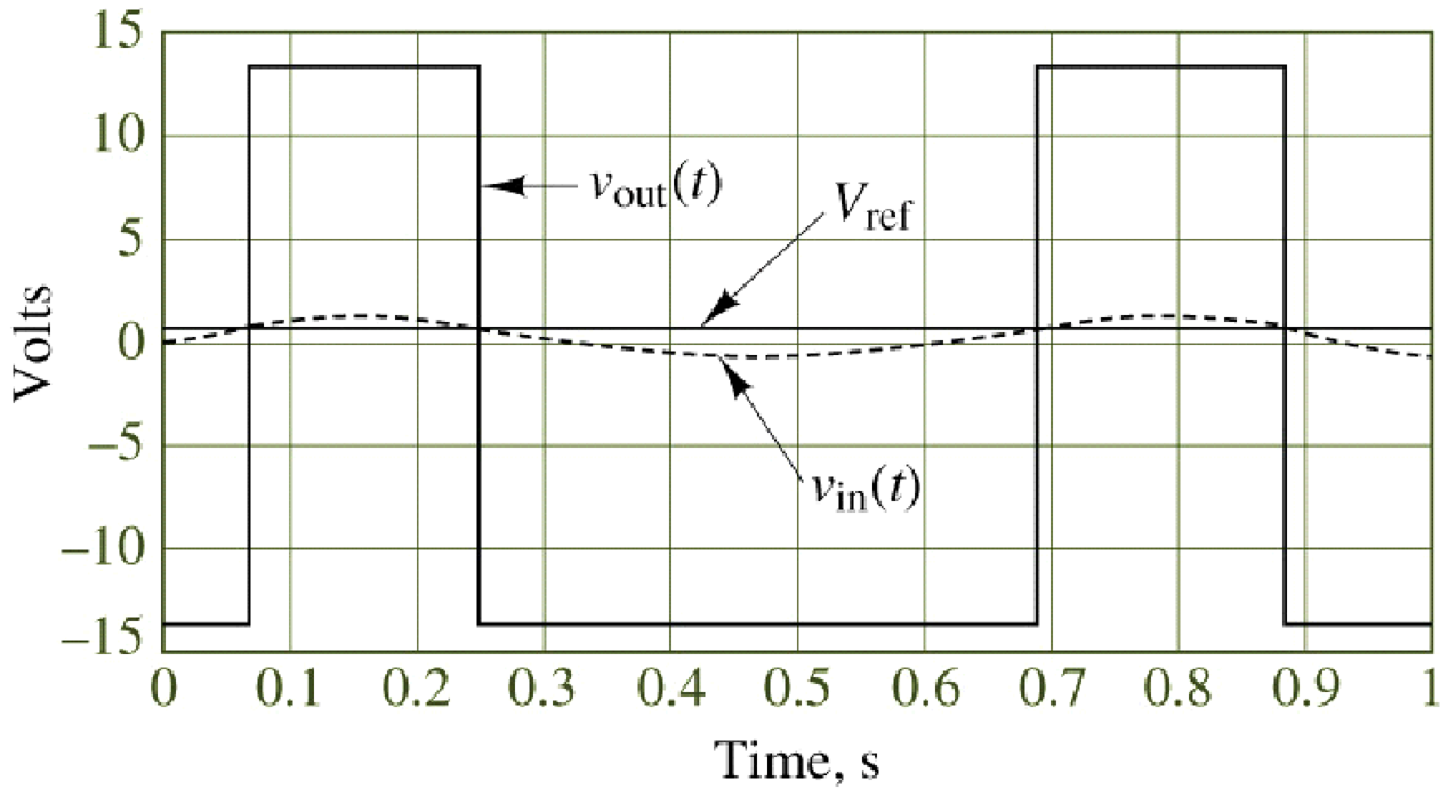


– Comparator with Offset

- A simple modification of the comparator circuit consists of connecting a fixed reference voltage to one of the input terminals; the effect of the reference voltage is to raise or lower the voltage level at which the comparator will switch from one extreme to the other.

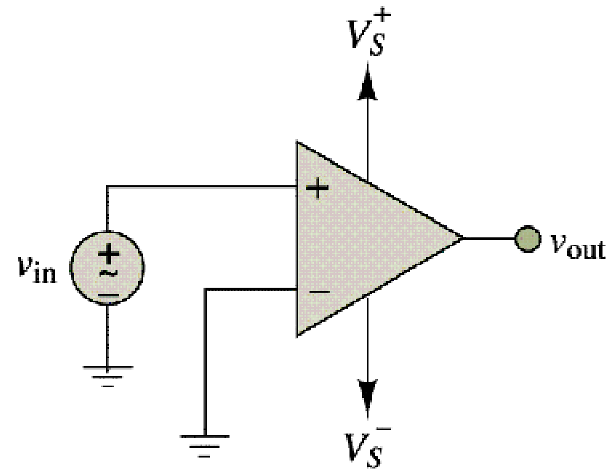


– Below is the waveform of a comparator with a reference voltage of 0.6 V and an input voltage of sin(ωt).

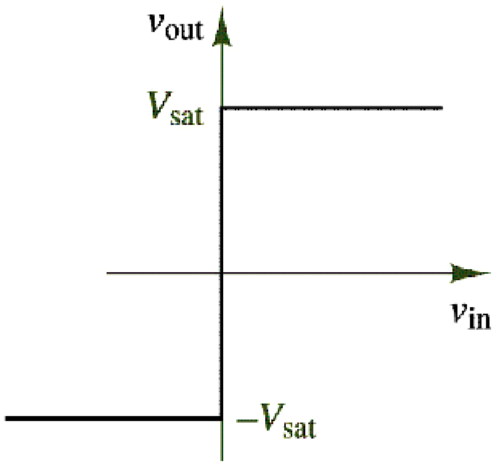


– Note that the comparator output is no longer a symmetric square wave.

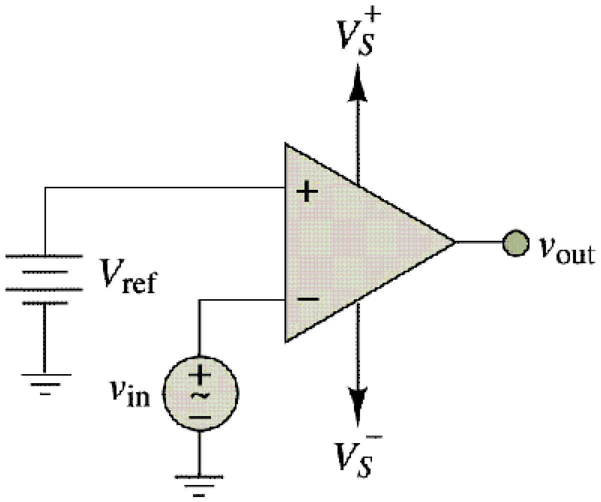
– Another useful interpretation of the op-amp comparator can be obtained by considering its input-output transfer characteristic.

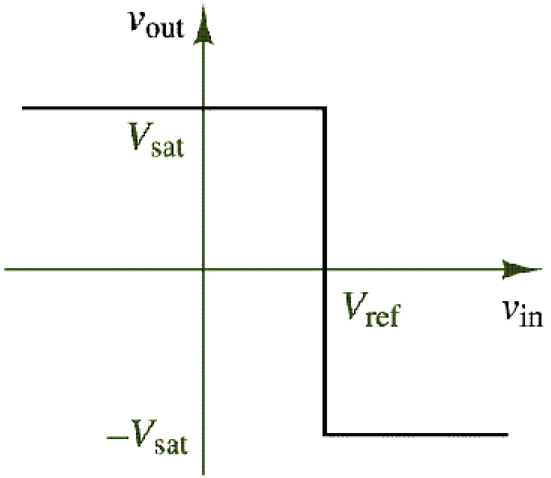


Non-Inverting Zero-Reference (no offset) Comparator often called a zero-crossing comparator



– Shown below is the transfer characteristic for a comparator of the inverting type with a nonzero reference voltage.



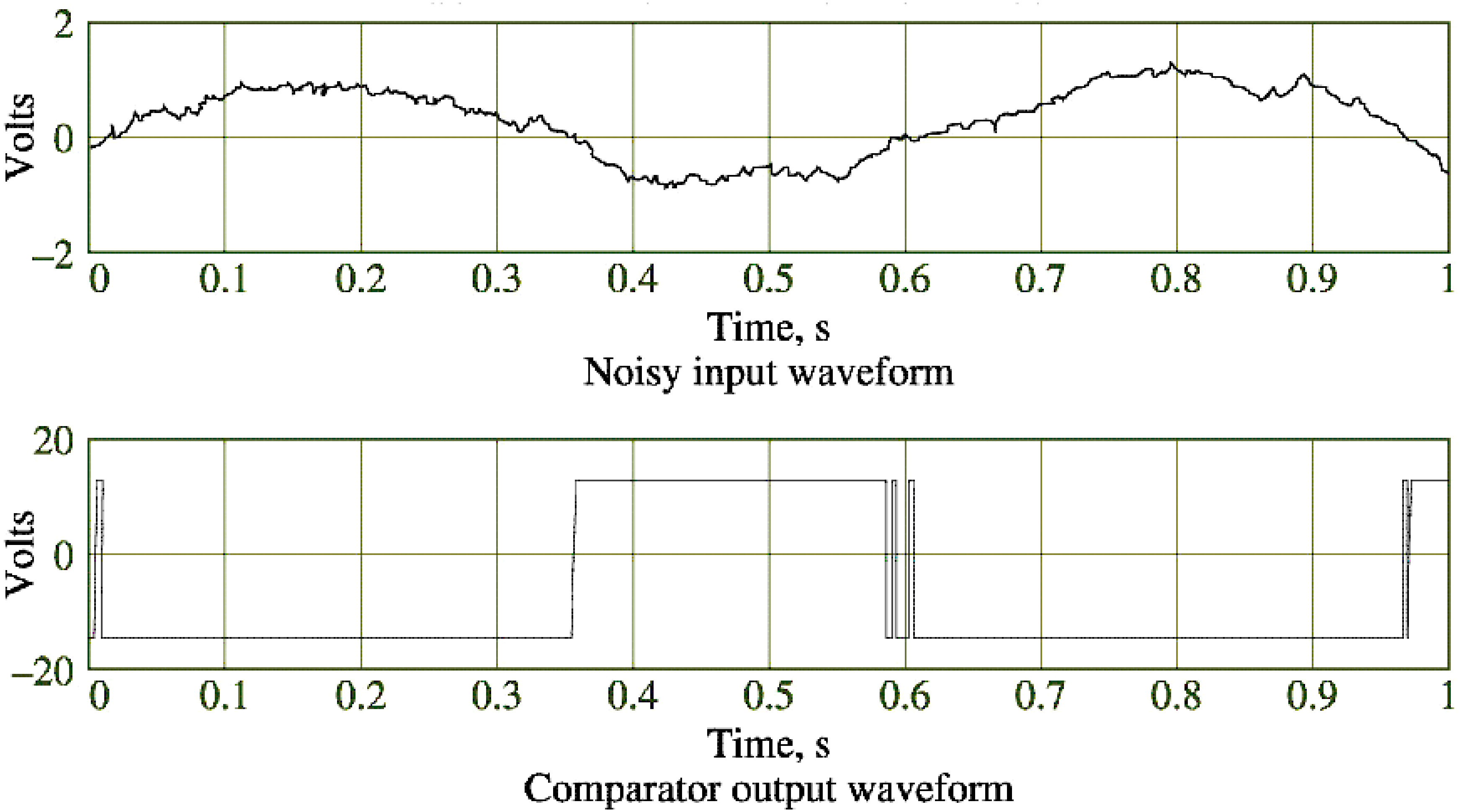


– Very often, in converting an analog signal to a binary representation, one would like to use voltage levels other than $\pm V_{sat}$. Commonly used voltage levels in this type of switching circuit are 0 and 5V.
– Special-purpose integrated-circuit packages are available that are specially designed to serve as comparators. These typically can accept relatively large inputs and have provision for selecting the desired reference voltage levels.
– An example is the LM311, which provides an open-collector output. The open-collector output allows the user to connect the output transistor to any supply voltage of choice by means of an external pull-up resistor, thus completing the output circuit.

– The actual value of the resistor is not critical, since the transistor is operated in the saturation mode; values between a few hundred and a few thousand ohms are typical.

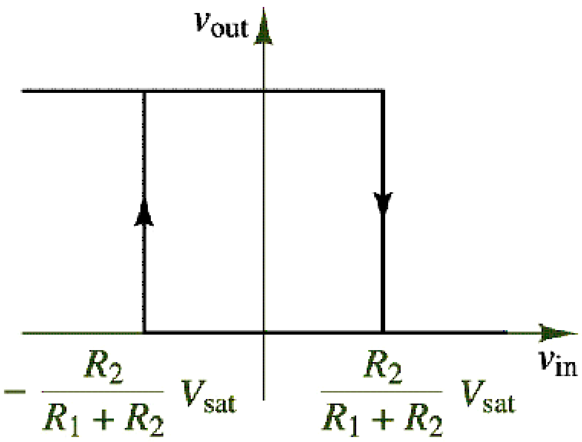
- Schmitt Trigger
  - One of the typical applications of the op-amp comparator is in detecting when an input voltage exceeds a present threshold voltage. The desired threshold is then represented by a DC reference V_{ref} connected to the non-inverting input, and the input voltage source is connected to the inverting input.
  - The presence of noise and a finite slew rate of practical op-amps requires special attention.
  - Two improvements are discussed:
    - How to improve the switching speed of the comparator.
    - How to design a circuit that can operate correctly even in the presence of noisy signals.

Comparator Response to Noisy Inputs

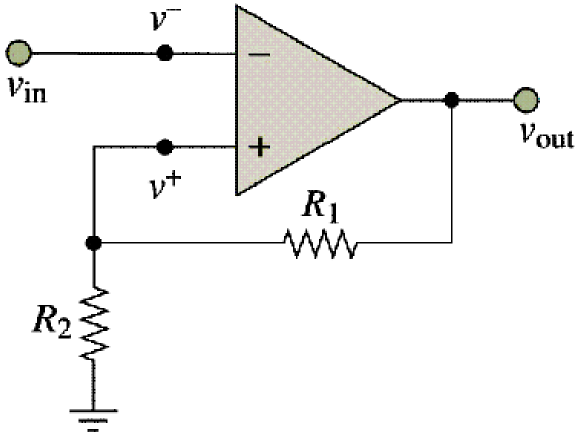


– One very effective way of improving the performance of the comparator is by introducing positive feedback. Positive feedback can increase the switching speed of the comparator and provide noise immunity at the same time.

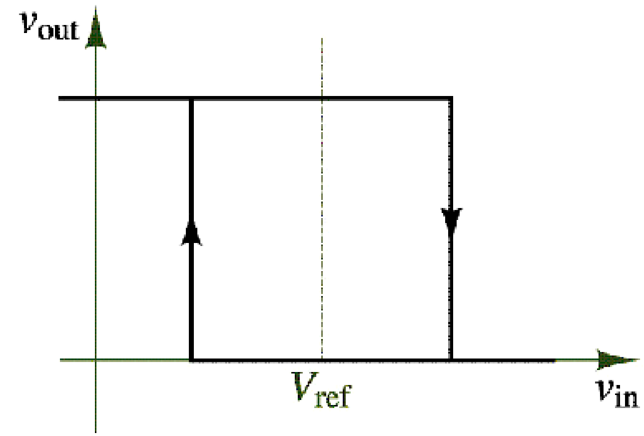
Transfer Characteristic
of the
Schmitt Trigger



Can you explain how this works?



– If it is desired to switch about a voltage other than zero, a reference voltage can also be connected to the non-inverting terminal.



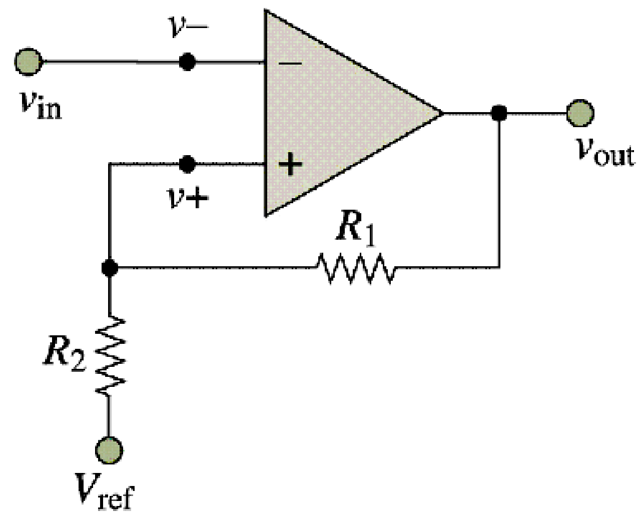
Switching levels for the Schmitt Trigger are:

$$V_{in} > \frac{R_2}{R_2 + R_1} V_{sat} + V_{ref} \frac{R_1}{R_2 + R_1}$$

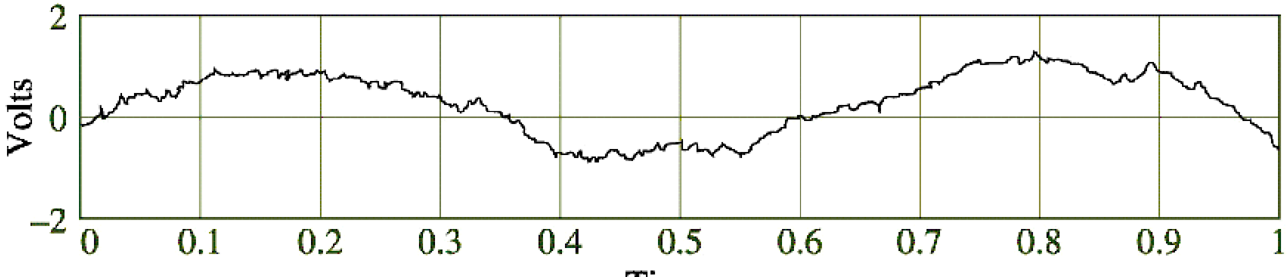
positive-going transition

$$V_{in} < -\frac{R_2}{R_2 + R_1} V_{sat} + V_{ref} \frac{R_1}{R_2 + R_1}$$

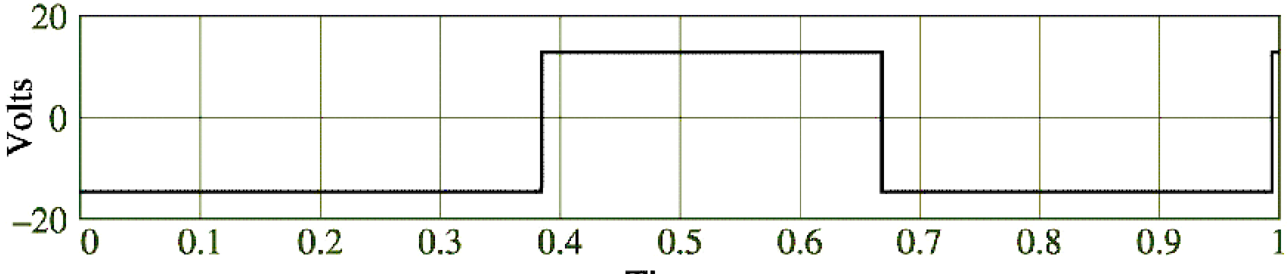
negative-going transition



- In effect, the Schmitt trigger provides a noise rejection range equal to $\pm V_{sat} [R_2 / (R_2 + R_1)]$ within which the comparator cannot switch.
- Thus if the noise amplitude is contained within this range, the Schmitt trigger will prevent multiple triggering.



Noisy input waveform



Schmitt trigger output waveform