Electromechanics applied to the:
- Magnetic Levitation System
- Brushed DC Motor

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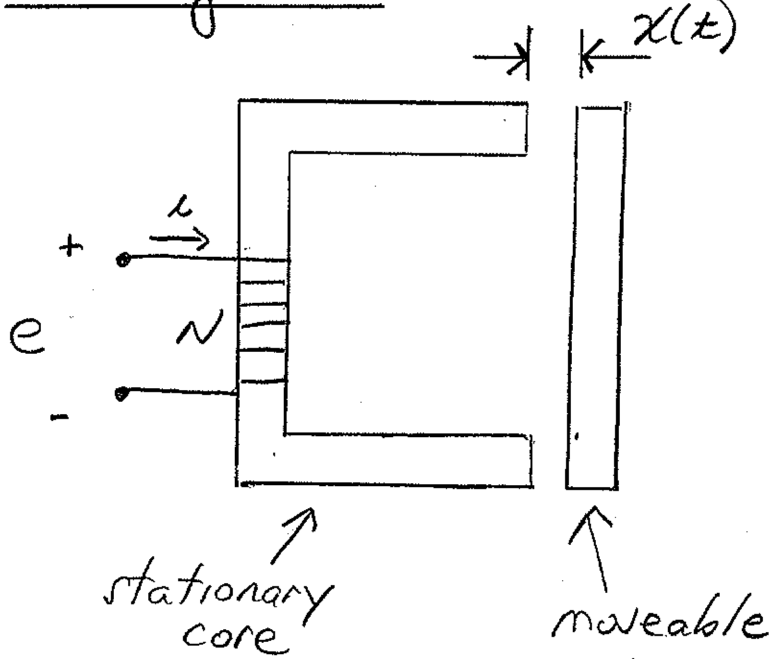
Background



1 DOF system – x motion (frictionless)

$$e = Ri + \frac{d\lambda}{dt}$$

Faraday's Law

voltage equation KVL

λ = flux linkages
$= N\phi$

$$\phi = \frac{Ni}{R}$$

Ni = magnetomotive force
R = reluctance
ϕ = magnetic flux

Linear System
- Neglect Saturation

(Also neglect any leakage flux.)

$$\lambda = N\phi = N\left(\frac{Ni}{R}\right) = \frac{N^2}{R} i = L i$$

L = inductance $= \dfrac{N^2}{R}$

R = reluctance $= \dfrac{\ell}{\mu A}$

Here $R_{total} = R_{core} + R_{air\,gaps} + R_{bar}$

$$R_{core} + R_{bar} = \frac{\ell}{\mu_r \mu_0 A} \qquad R_{gap} = \frac{2x}{\mu_0 A_g}$$

Assume $A = A_g$

$$R_{total} = \frac{1}{\mu_0 A}\left(\frac{\ell}{\mu_r} + 2x\right)$$

$$L = \frac{N^2}{\frac{1}{\mu_0 A}\left(\frac{\ell}{\mu_r} + 2x\right)}$$

$L = L(x)$

$\lambda(i, x) = L(x)\, i$

$$\frac{d\lambda}{dt} = \frac{\partial \lambda}{\partial i}\frac{di}{dt} + \frac{\partial \lambda}{\partial x}\frac{dx}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

Voltage equation

$$e = Ri + L(x)\frac{di}{dt} + i\,\frac{dL(x)}{dx}\frac{dx}{dt}$$

nonlinear differential equation

self-inductance voltage term

speed-voltage term common to all electromechanical-energy-conversion systems; it is responsible for energy transfer to and from the mechanical system by the electrical system

$$L(x) = \frac{N^2}{\frac{1}{\mu_0 A}\left(\frac{l}{\mu_r} + 2x\right)}$$

$$= \frac{K}{K_0 + x}$$

$$K = \frac{N^2 \mu_0 A}{2}$$

$$K_0 = \frac{l}{2\mu_r}$$

$$L(x) \approx \frac{K}{x}$$

for $x > 0$

System is magnetically linear:

$$W_{field}(i, x) = \frac{1}{2} L(x)\, i^2$$

$$fe(i, x) = \frac{\partial W(i, x)}{\partial x} = \frac{1}{2} i^2 \frac{\partial L(x)}{\partial x}$$

$$= -\frac{K i^2}{2x^2}$$

The electromagnetic force is set up so as to minimize the reluctance (maximize the inductance) of the magnetic system.

The force fe is always negative; it pulls the moving member to the stationary member.