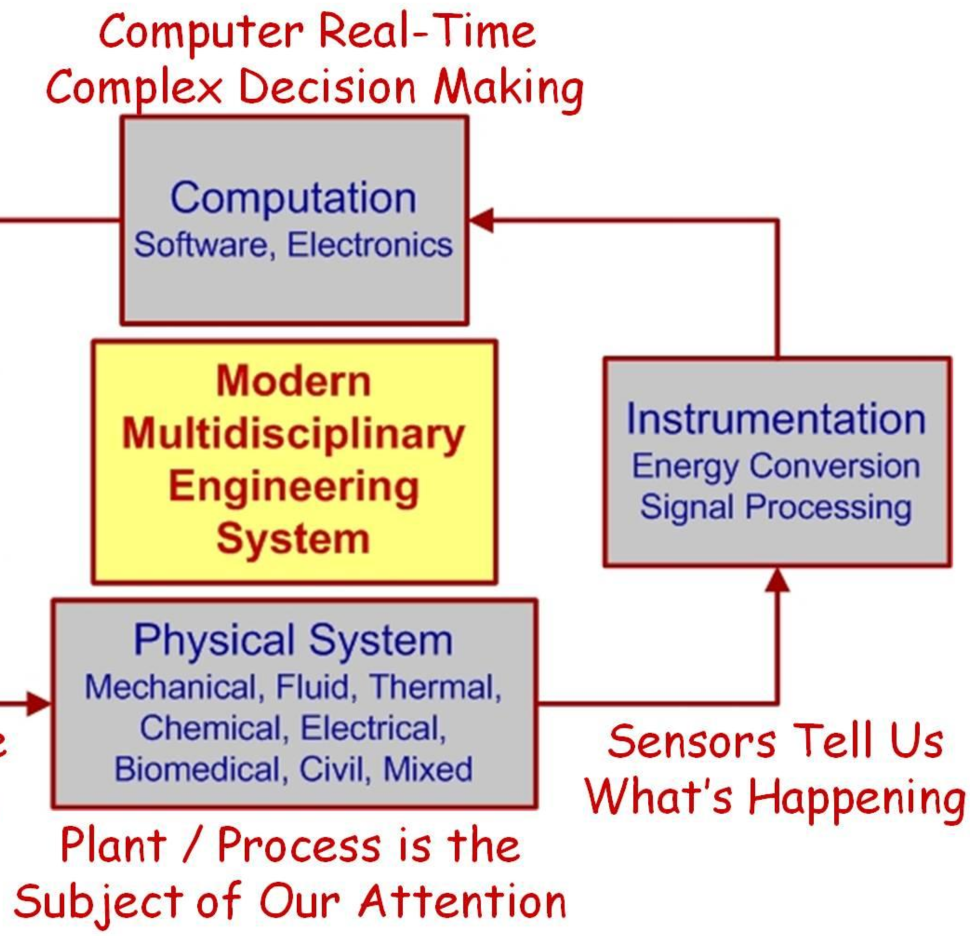
Mechanical System Modeling



References for Mechanical Systems

- *System Dynamics*, E. Doebelin, Marcel Dekker, 1998. (This is the finest reference on system dynamics available; many figures in these notes are taken from this reference.)
- *Modeling, Analysis, and Control of Dynamic Systems*, W. Palm, 2nd Edition, Wiley, 1999.
- *Vector Mechanics for Engineers: Dynamics*, 7th Edition, F. Beer, E.R. Johnston, and W. Clausen, McGraw Hill, 2004.

Mechanical System Elements

- Three Basic Mechanical Elements
  - Spring (elastic) element
  - Damper (frictional) element
  - Mass (inertia) element
- Translational and Rotational versions
- These are passive (non-energy producing) devices
- Driving Inputs
  - force and motion sources which cause elements to respond

- Each of the elements has one of two possible energy behaviors:
  - – stores all the energy supplied to it
  - – dissipates all energy into heat by some kind of "frictional" effect
    - Spring stores energy as potential energy
    - Mass stores energy as kinetic energy
    - Damper dissipates energy into heat
- Dynamic Response of each element is important
  - – step response
  - – frequency response

Spring Element

- Real-world design situations
- Real-world spring is neither pure nor ideal
- Real-world spring has inertia and friction
- Pure spring has only elasticity - it is a mathematical model, not a real device
- Some dynamic operation requires that spring inertia and/or damping not be neglected
- Ideal spring: linear
- Nonlinear behavior may often be preferable and give significant performance advantages

- Device can be pure without being ideal (e.g., nonlinear spring with no inertia or damping)
- Device can be ideal without being pure (e.g., device which exhibits both linear springiness and linear damping)
- <u>Pure and ideal spring element</u>

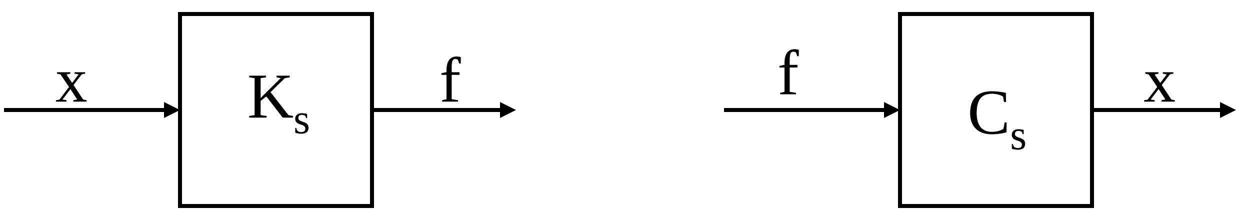
$$f = K_s\left(x_1 - x_2\right) = K_s x$$

$$T = K_s\left(\theta_1 - \theta_2\right) = K_s \theta$$

- K_s = spring stiffness (N/m or N-m/rad)
- $1/K_s = C_s$ = compliance (softness parameter)

$$x = C_s f$$

$$\theta = C_s T$$



- Energy stored in a spring $$E_s = \frac{C_s f^2}{2} = \frac{K_s x^2}{2}$$
- Dynamic Response: Zero-Order Dynamic System Model
  – Step Response
  – Frequency Response
- Real springs will not behave exactly like the pure/ideal element. One of the best ways to measure this deviation is through frequency response.

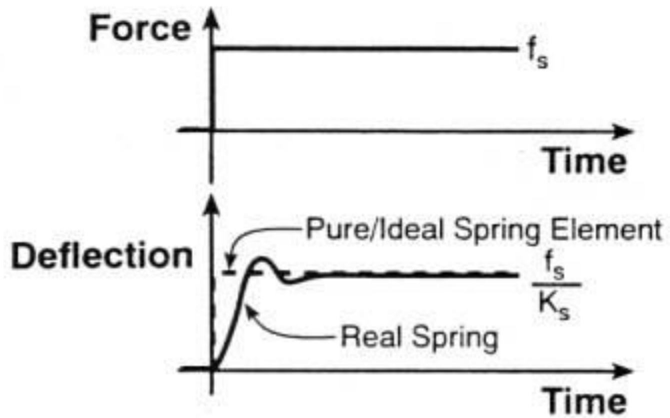
Spring Element

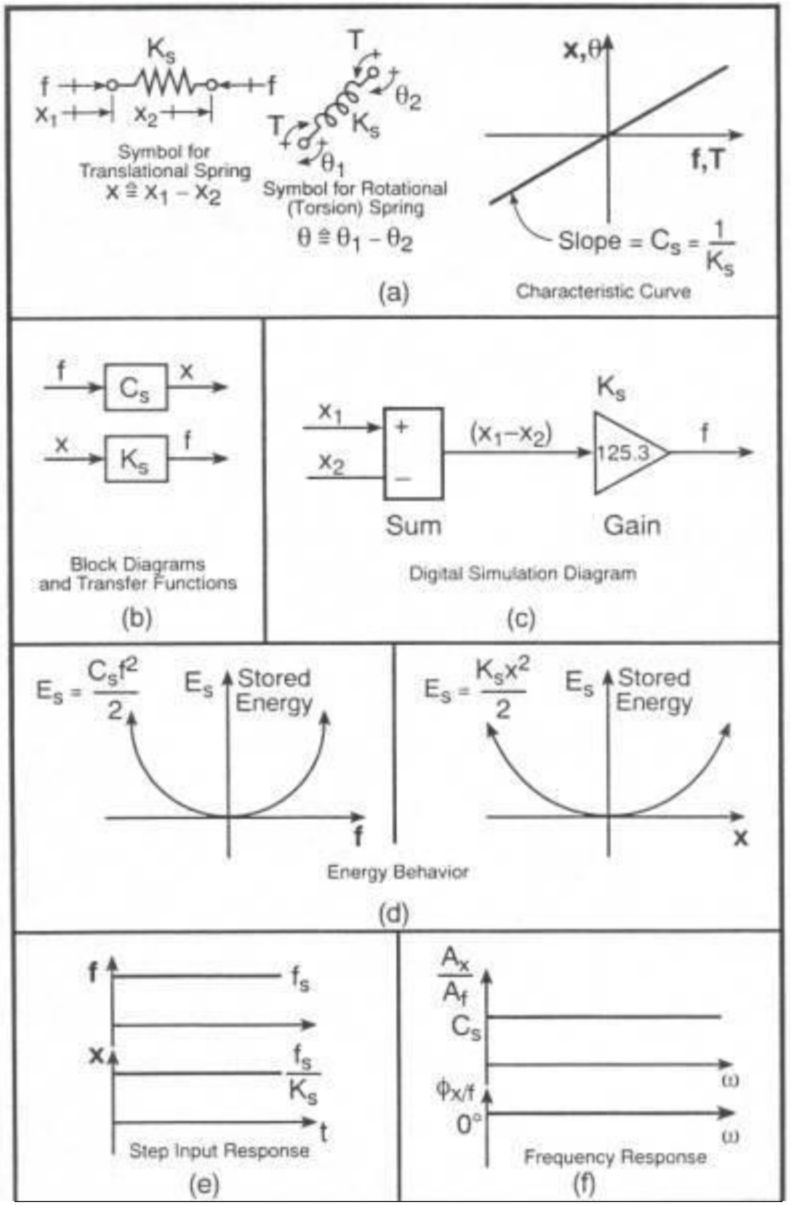
Differential Work Done

$$= (f)dx = (K_s x)dx$$

Total Work Done

$$= \int_0^{x_0} (K_s x)dx = \frac{K_s x_0^2}{2} = \frac{C_s f_0^2}{2}$$

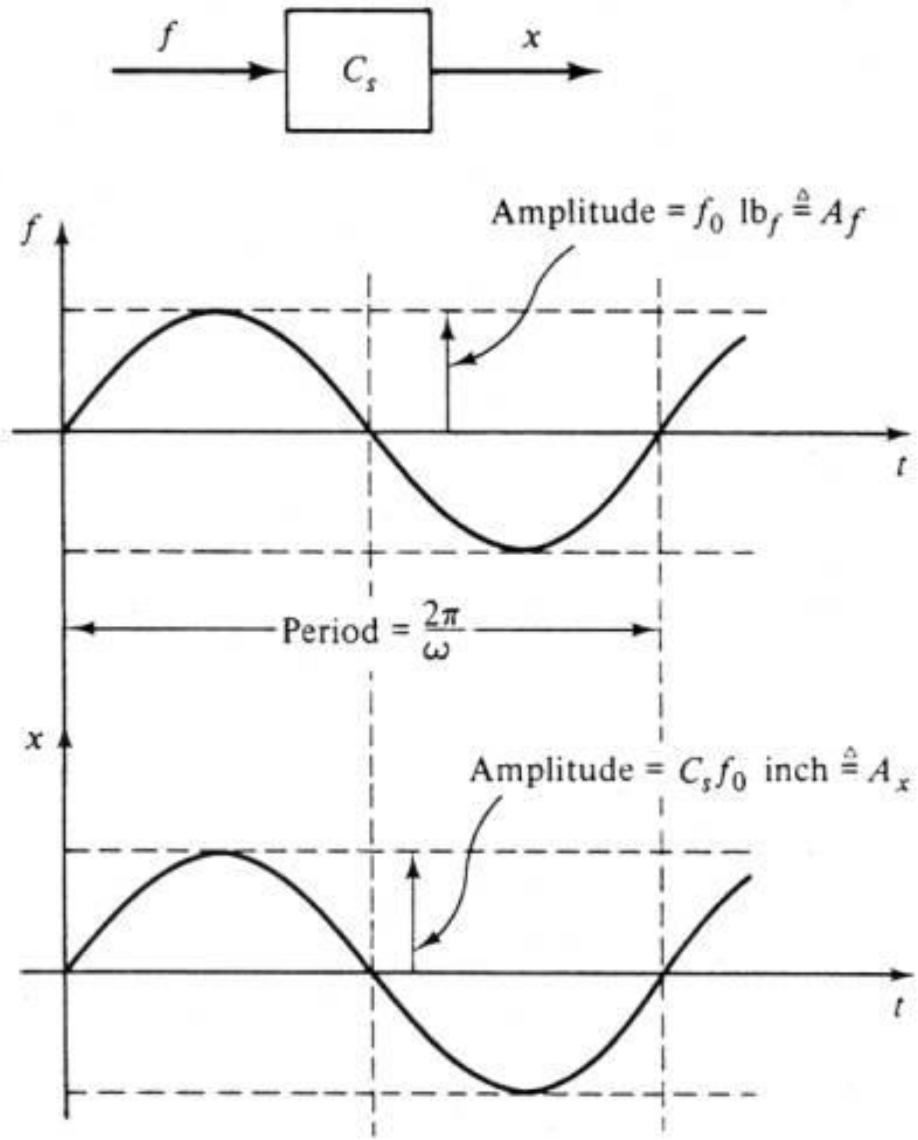




Frequency Response of Spring Elements

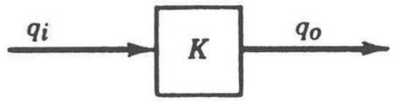
$$f = f_0 \sin(\omega t)$$

$$x = C_s f_0 \sin(\omega t)$$

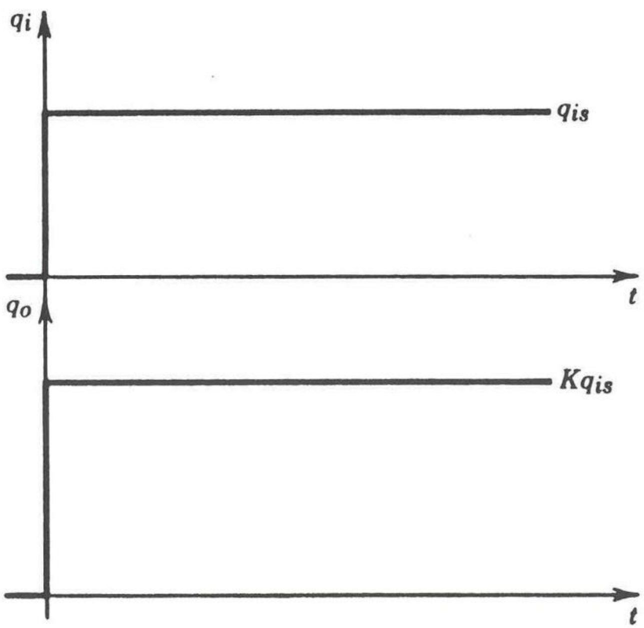


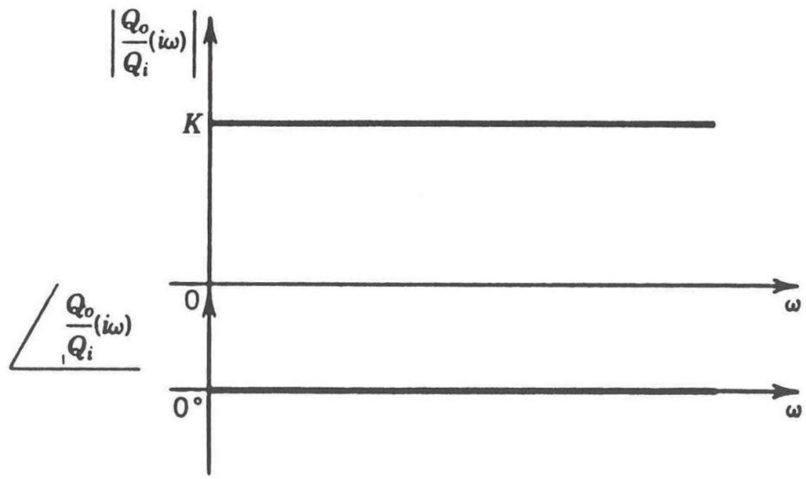
Zero-Order Dynamic System Model

$$q_o = K\, q_i$$



$$\frac{Q_o}{Q_i}(s) = K \qquad \frac{Q_o}{Q_i}(i\omega) = K\angle 0^\circ$$

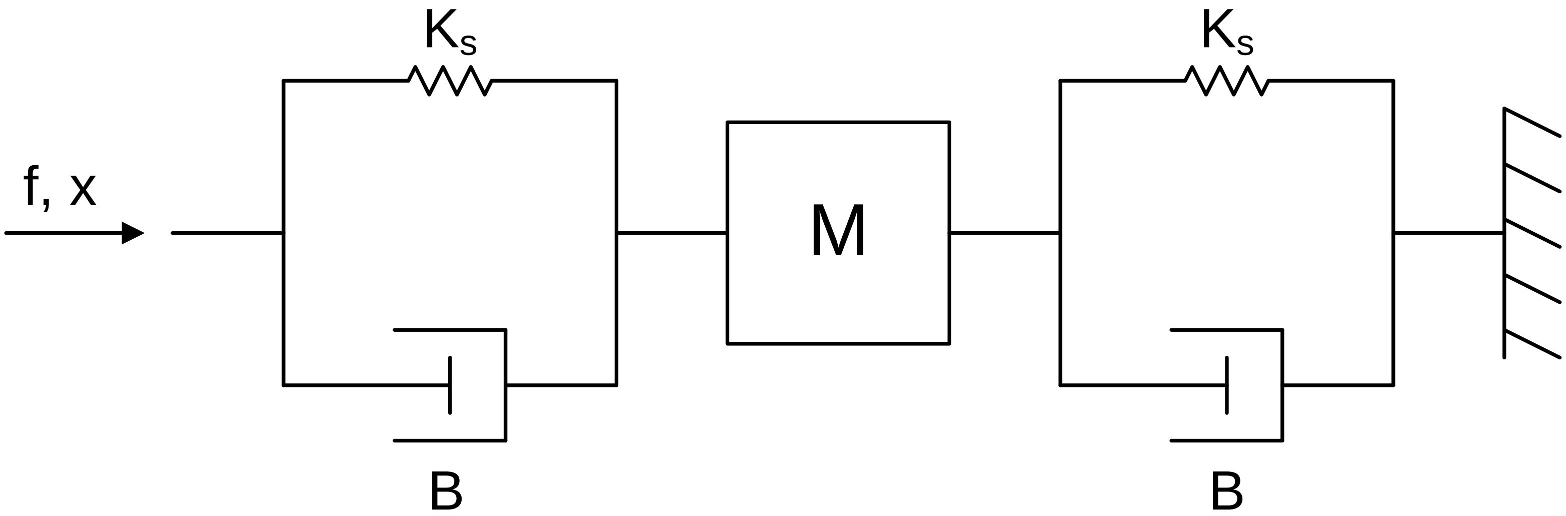




Step Response

Frequency Response

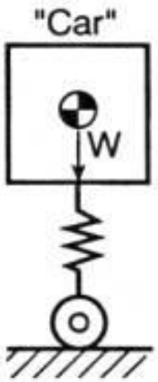
More Realistic Lumped-Parameter Model for a Spring



Linearization for a Nonlinear Spring

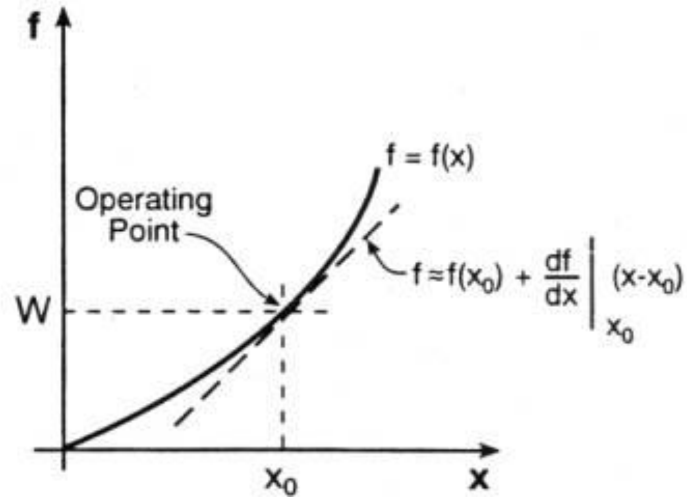
$$y = f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0}(x - x_0) + \left.\frac{d^2f}{dx^2}\right|_{x=x_0}\frac{(x - x_0)^2}{2!} + \cdots$$

$$y \approx y_0 + \left.\frac{df}{dx}\right|_{x=x_0}(x - x_0)$$

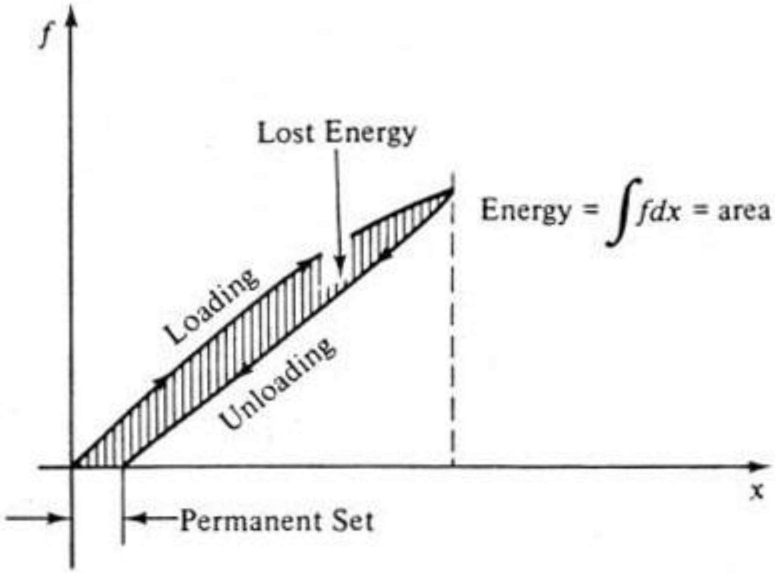


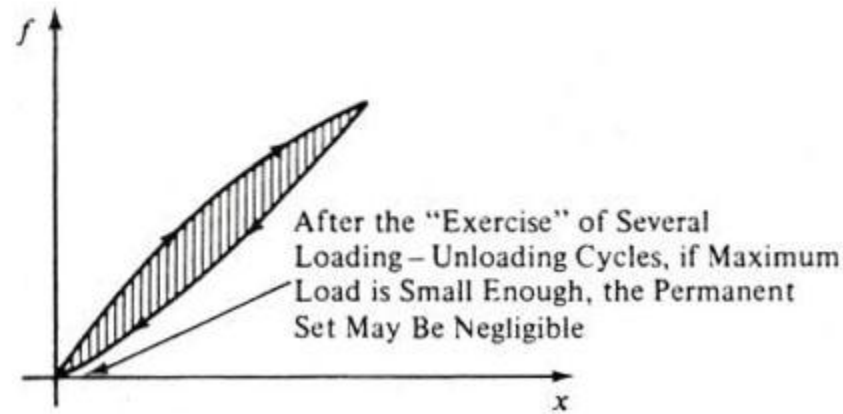
$$y - y_0 \approx + \left.\frac{df}{dx}\right|_{x=x_0}(x - x_0)$$

$$\hat{y} = K\hat{x}$$

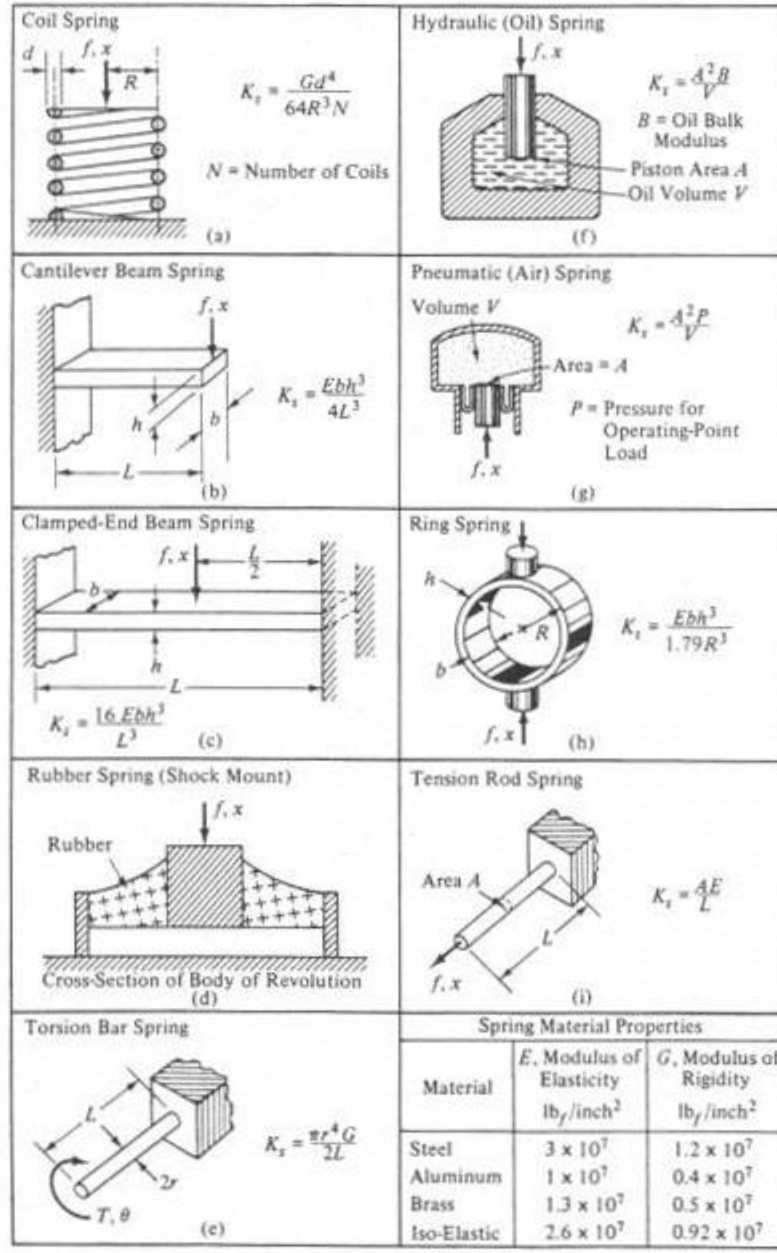


- Real Springs
  - nonlinearity of the force/deflection curve
  - noncoincidence of the loading and unloading curves (The 2nd Law of Thermodynamics guarantees that the area under the loading *f vs. x* curve must be greater than that under the unloading *f vs. x* curve. It is impossible to recover 100% of the energy put into any system.)





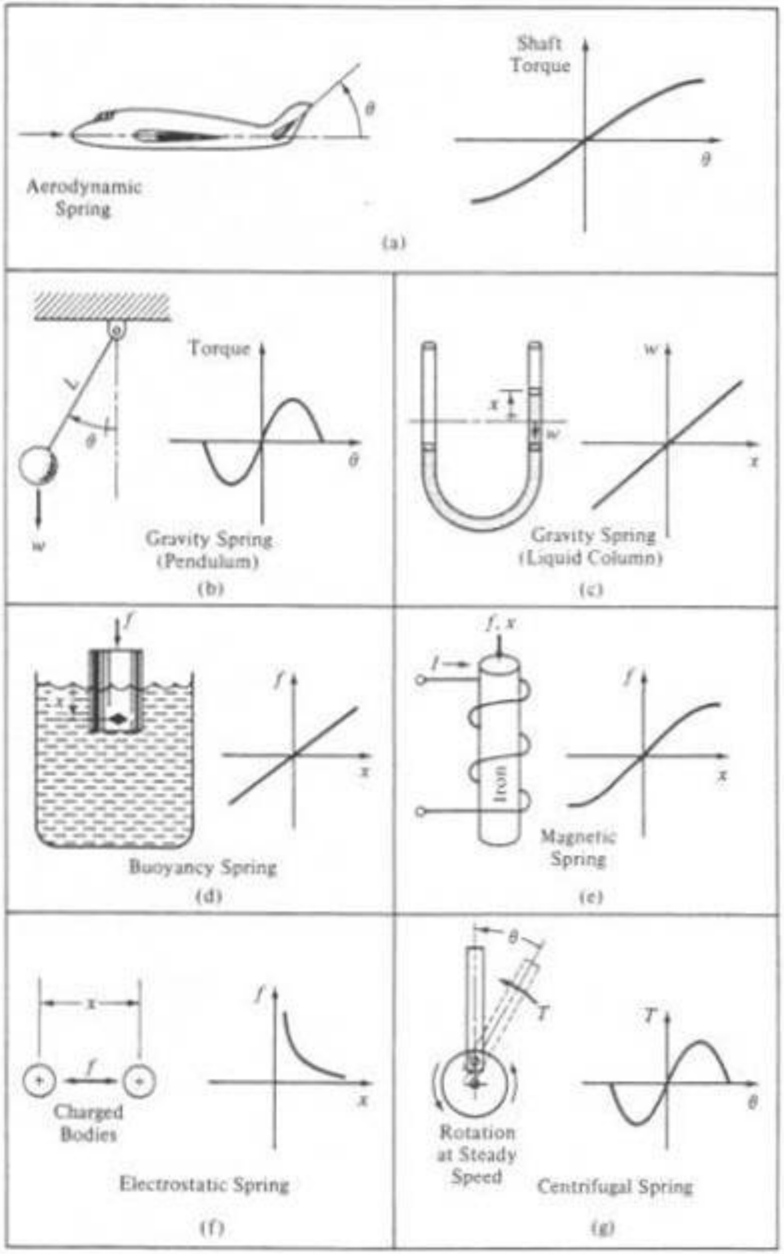
- Several Types of Practical Springs:
  - coil spring
  - hydraulic (oil) spring
  - cantilever beam spring
  - pneumatic (air) spring
  - clamped-end beam spring
  - ring spring
  - rubber spring (shock mount)
  - tension rod spring
  - torsion bar spring



Spring Material Properties

Material	E, Modulus of Elasticity lb$_f$/inch^2	G, Modulus of Rigidity lb$_f$/inch^2
Steel	3×10^7	1.2×10^7
Aluminum	1×10^7	0.4×10^7
Brass	1.3×10^7	0.5×10^7
Iso-Elastic	2.6×10^7	0.92×10^7

- Spring-like Effects in Unfamiliar Forms
  - aerodynamic spring
  - gravity spring (pendulum)
  - gravity spring (liquid column)
  - buoyancy spring
  - magnetic spring
  - electrostatic spring
  - centrifugal spring



Damper Element

- A pure damper dissipates all the energy supplied to it, i.e., converts the mechanical energy to thermal energy.
- Various physical mechanisms, usually associated with some form of friction, can provide this dissipative action, e.g.,
  - Coulomb (dry friction) damping
  - Material (solid) damping
  - Viscous damping

- Pure / ideal damper element provides viscous friction.
- All mechanical elements are defined in terms of their force/motion relation. (Electrical elements are defined in terms of their voltage/current relations.)
- Pure / Ideal Damper
  - Damper force or torque is directly proportional to the relative velocity of its two ends.

$$f = B\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) = B\frac{dx}{dt} \qquad T = B\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) = B\frac{d\theta}{dt}$$

- Forces or torques on the two ends of the damper are exactly equal and opposite at all times (just like a spring); pure springs and dampers have no mass or inertia. This is NOT true for real springs and dampers.
- Units for *B* to preserve physical meaning:
  - N/(m/sec)
  - (N-m)/(rad/sec)
- Transfer Function

Differential Operator Notation

$$Dx = \frac{dx}{dt} \qquad D^2x = \frac{d^2x}{dt^2}$$

$$\frac{x}{D} = \int (x)dt \qquad \frac{x}{D^2} = \int \left[\int (x) dt \right] dt$$

– Operational Transfer Functions

$$\left.\begin{aligned} f &= BDx \\ T &= BD\theta \end{aligned}\right\} \qquad \begin{aligned} &\frac{f}{x}(D) = BD \qquad &&\frac{T}{\theta}(D) = BD \\ &\frac{x}{f}(D) = \frac{1}{BD} \qquad &&\frac{\theta}{T}(D) = \frac{1}{BD} \end{aligned}$$

- We assume the initial conditions are zero.

– Damper element dissipates into heat all mechanical energy supplied to it.

$$\text{Power} = (\text{force})(\text{velocity}) = f\left(\frac{dx}{dt}\right) = B\left(\frac{dx}{dt}\right)^2$$

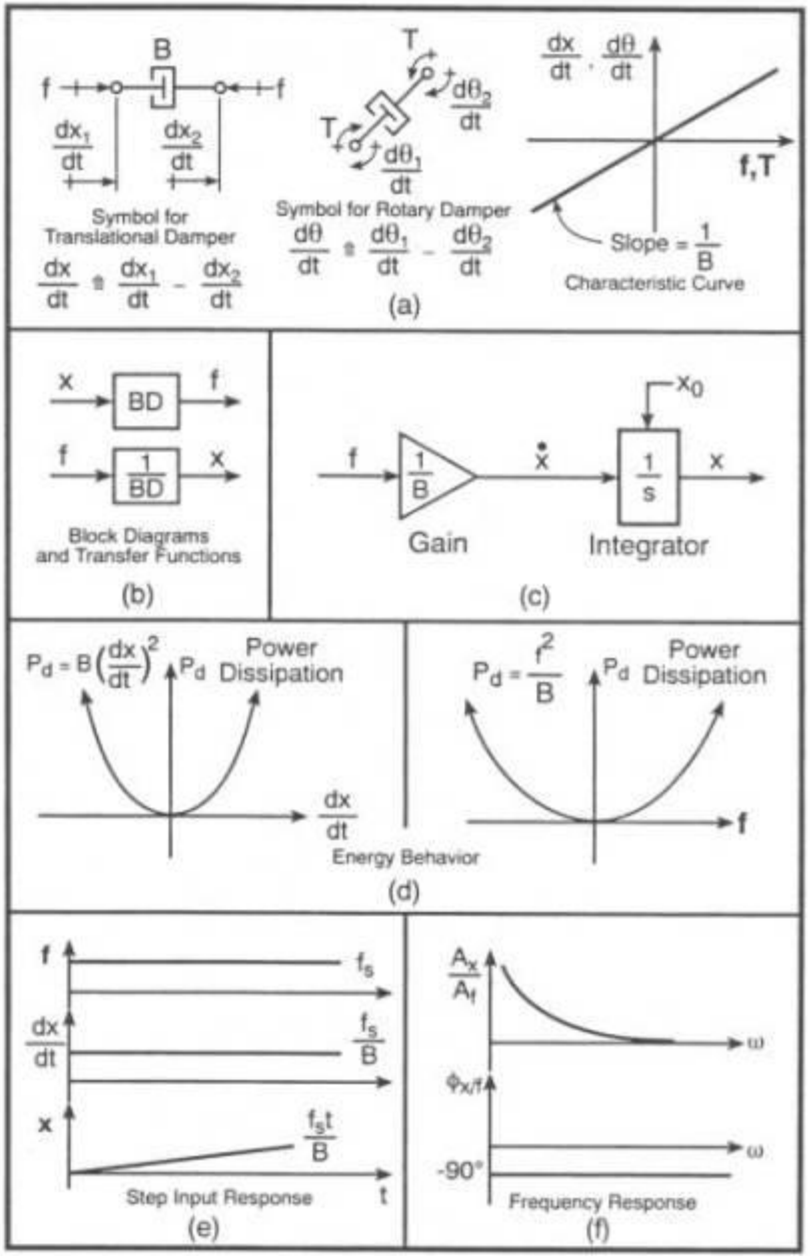
- Force applied to damper causes a velocity in same direction.

- Power input to the device is positive since the force and velocity have the same sign.
- It is impossible for the applied force and resulting velocity to have opposite signs.
- Thus, a damper can never supply power to another device; Power is always positive.
- A spring absorbs power and stores energy as a force is applied to it, but if the force is gradually relaxed back to zero, the external force and the velocity now have opposite signs, showing that the spring is delivering power.
- Total Energy Dissipated

$$\int (P)\,dt = \int B\left(\frac{dx}{dt}\right)^2 dt = \int B\left(\frac{dx}{dt}\right)dx = \int (f)\,dx$$

Damper Element

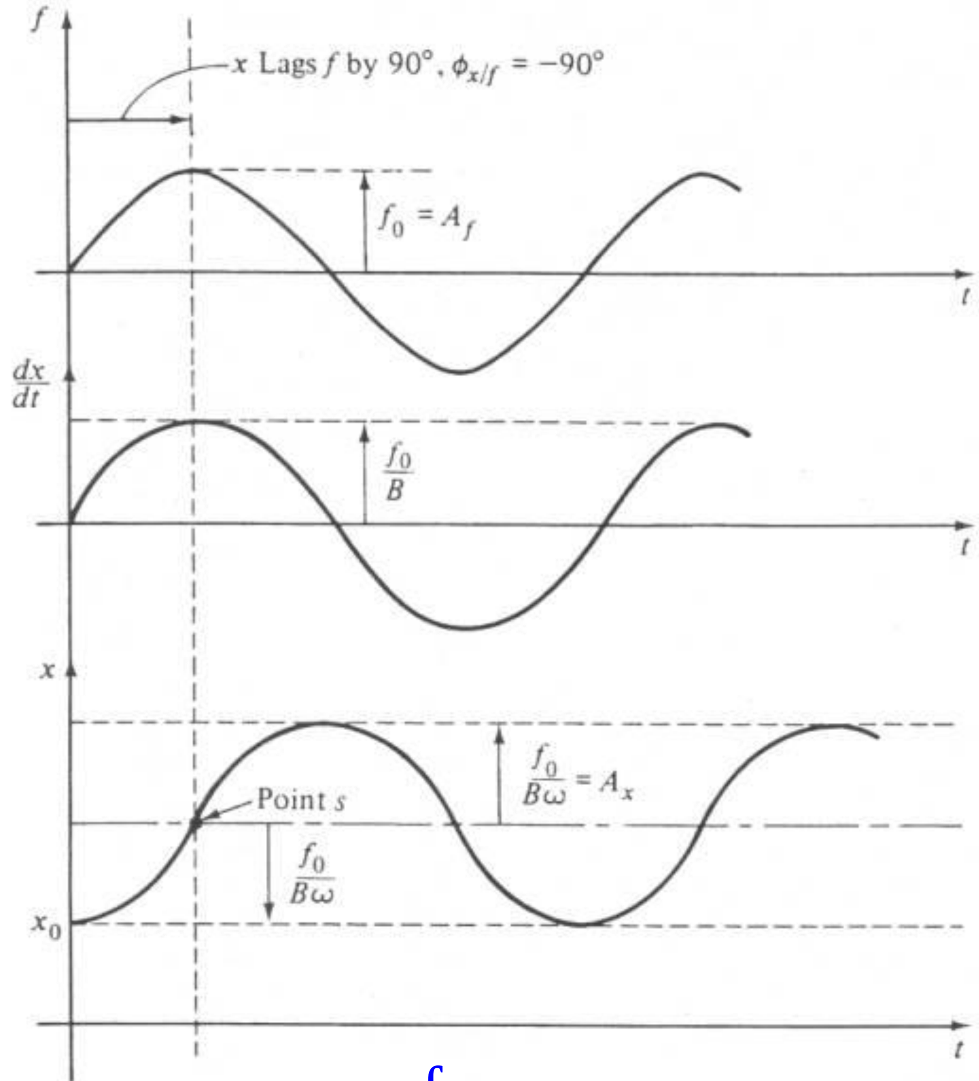
Step Input Force causes instantly (a pure damper has no inertia) a Step of dx/dt and a Ramp of x



Frequency Response of Damper Elements

$$f = f_0 \sin(\omega t) = B\frac{dx}{dt}$$

$$x - x_0 = \frac{1}{B}\int_0^t f_0 \sin(\omega t)\,dt = \frac{f_0}{B\omega}\left[1 - \cos(\omega t)\right]$$



$$\frac{A_x}{A_f} = \frac{\frac{f_0}{B\omega}}{f_0} = \frac{1}{B\omega}$$

- <u>Sinusoidal Transfer Function</u>

$$\frac{x}{f}(D) = \frac{1}{BD}$$

$$D \Rightarrow i\omega$$

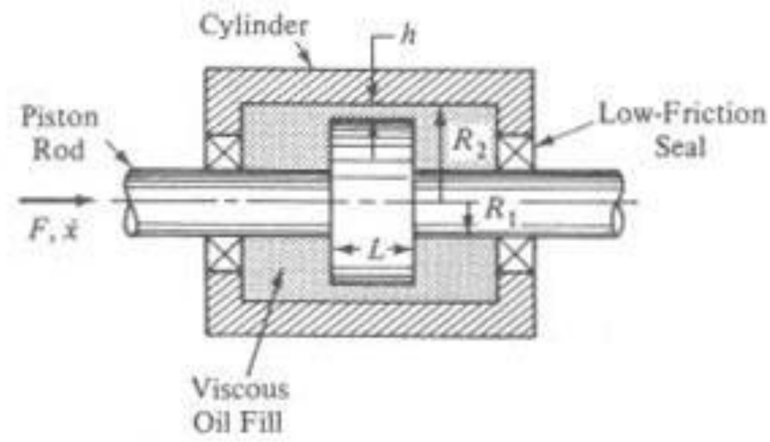
$$\frac{x}{f}(i\omega) = \frac{1}{i\omega B} = M\angle\phi$$

  - M is the amplitude ratio of output over input
  - φ is the phase shift of the output sine wave with respect to the input sine wave (positive if the output leads the input, negative if the output lags the input)

$$\frac{x}{f}(i\omega) = \frac{1}{i\omega B} = M\angle\phi = \frac{1}{B\omega}\angle -90^{\circ}$$

- Real Dampers
  - A damper element is used to model a device designed into a system (e.g., automotive shock absorbers) or for unavoidable parasitic effects (e.g., air drag).
  - To be an energy-dissipating effect, a device must exert a force opposite to the velocity; power is always negative when the force and velocity have opposite directions.
  - Let's consider examples of real *intentional* dampers.

Viscous (Piston/Cylinder) Damper



A relative velocity between the cylinder and piston forces the viscous oil through the clearance space *h*, shearing the fluid and creating a damping force.

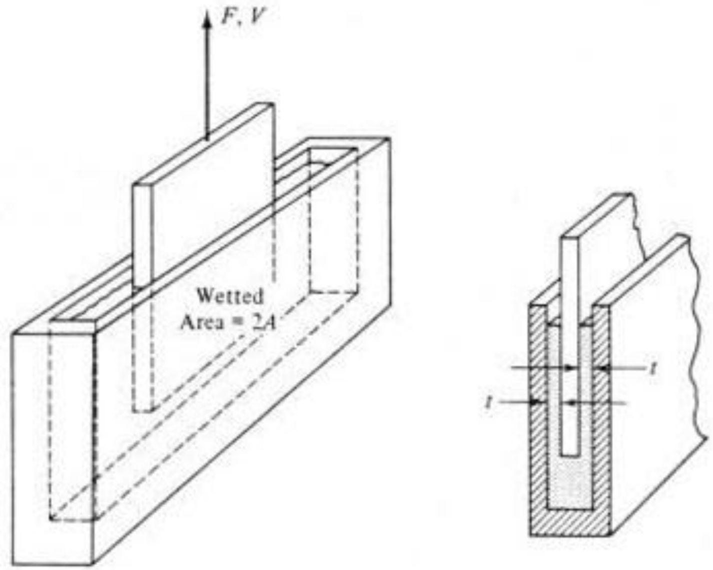
μ = fluid viscosity

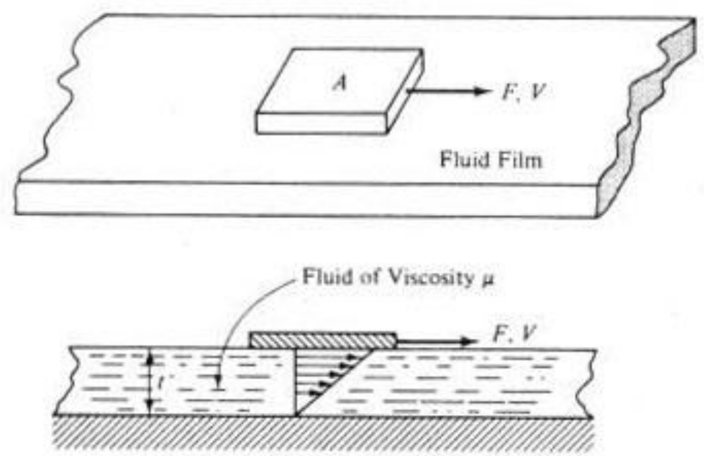
$$B = \frac{6\pi\mu L}{h^3}\left[\left(R_2 - \frac{h}{2}\right)^2 - R_1^2\right]\left[\frac{R_2^2 - R_1^2}{R_2 - \frac{h}{2}} - h\right]$$

Simple Shear Damper and Viscosity Definition

$$F = \frac{2A\mu}{t} V$$

$$B = \frac{F}{V} = \frac{2A\mu}{t}$$





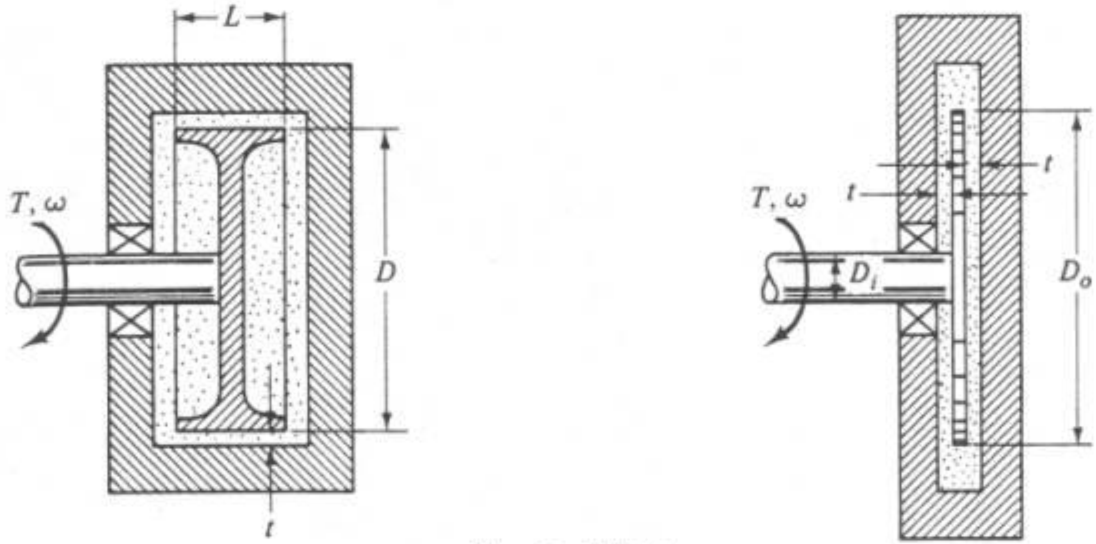
$$\mu = \text{fluid viscosity}$$

$$= \frac{\text{shearing stress}}{\text{velocity gradient}} = \frac{F/A}{V/t}$$

Examples of Rotary Dampers

$$B = \frac{\pi D_0^4 \mu}{16t}$$

$$B = \frac{\pi D^3 L \mu}{4t}$$



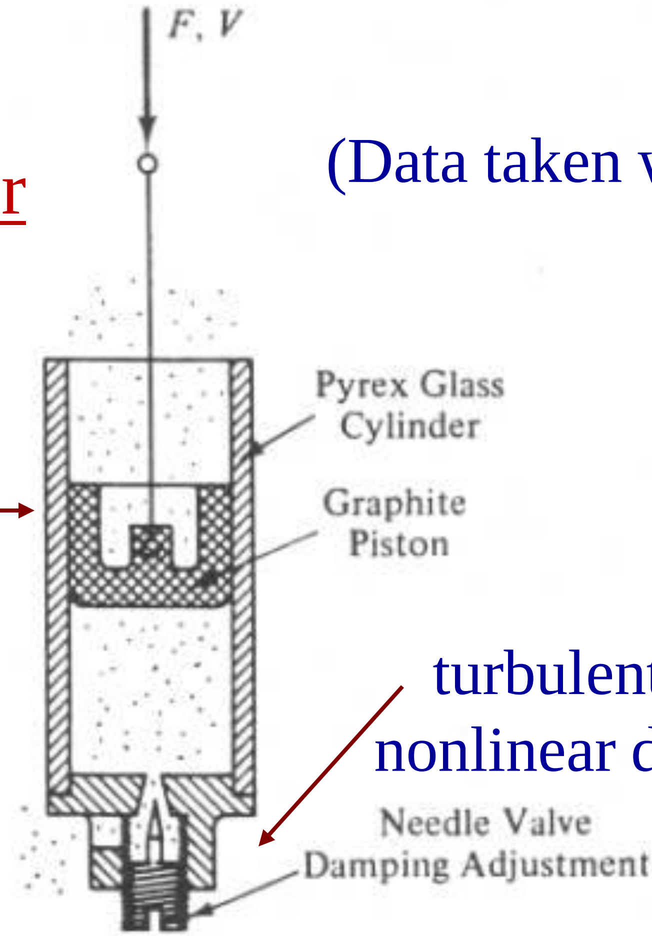
(a)

Damping Effects are Assumed to be Confined to the Gaps of Width t

(b)

Commercial Air Damper

(Data taken with valve shut)



laminar flow
linear damping

F lb_f	V inch/sec
0.0130	0.0104
0.0632	0.0526
0.1086	0.0935
0.1582	0.1389
0.2040	0.1755
0.2560	0.2170

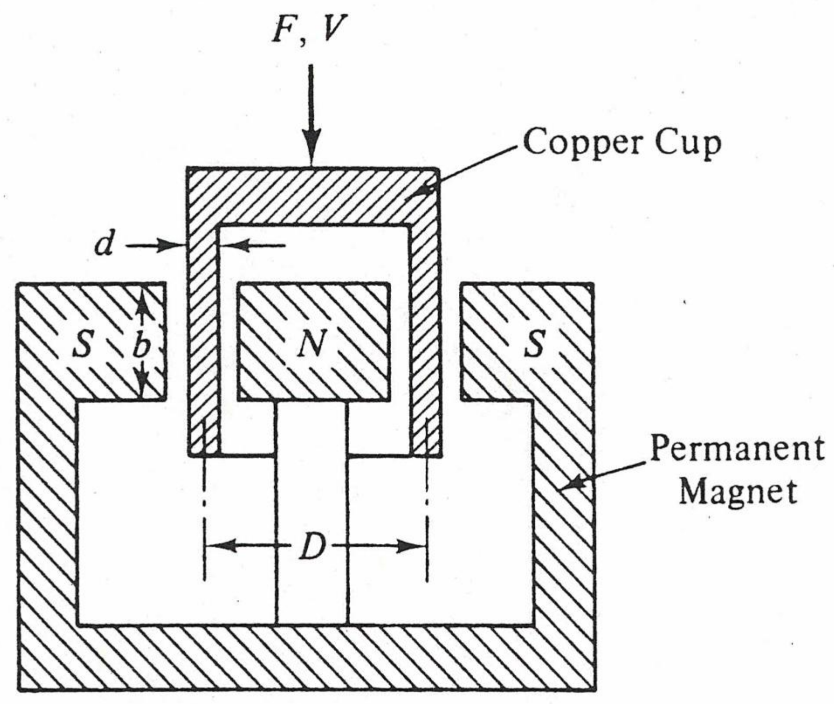
turbulent flow
nonlinear damping

Air Damper

- much lower viscosity
- less temperature dependent
- no leakage or sealing problem

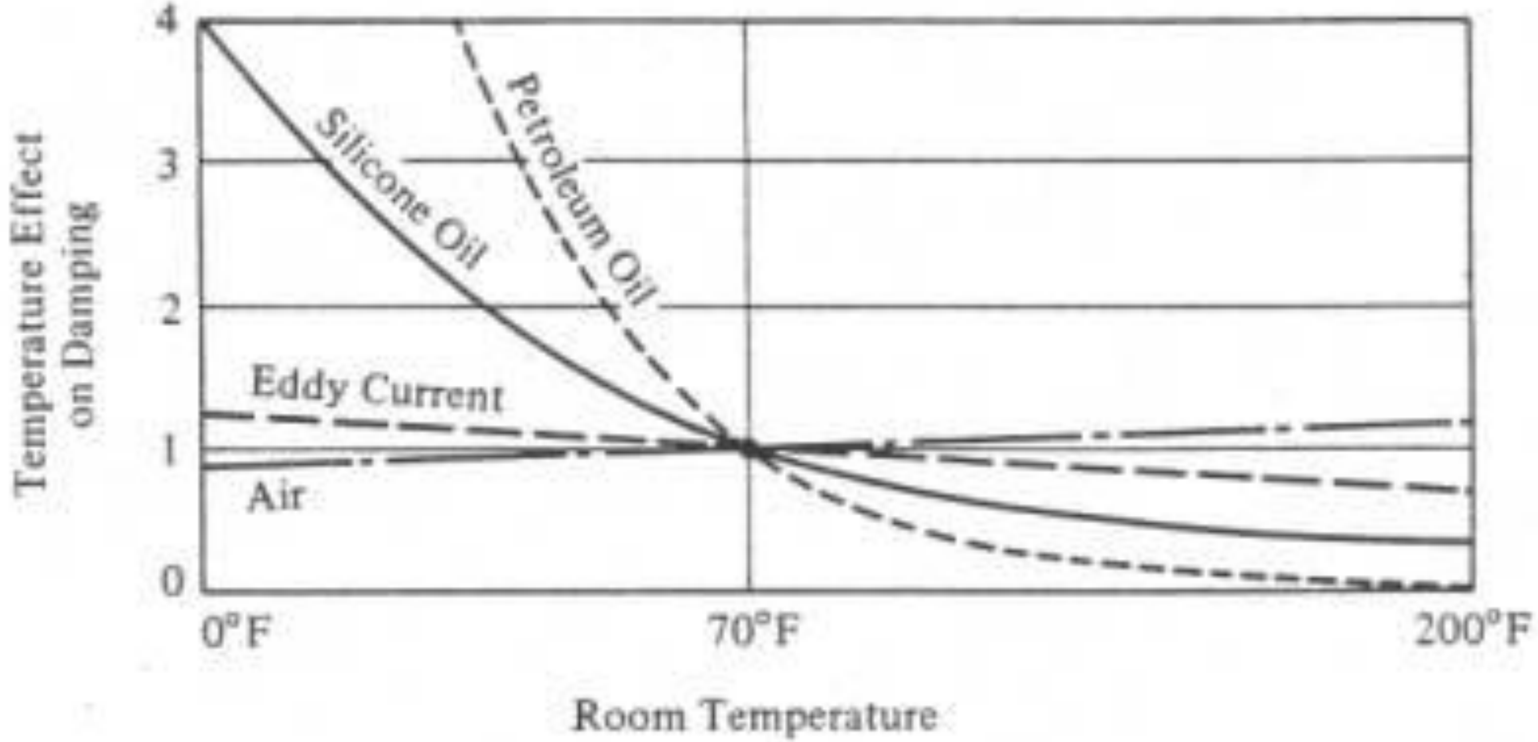
- Motion of the conducting cup in the magnetic field generates a voltage in the cup.
- A current is generated in the cup's circular path.
- A current-carrying conductor in a magnetic field experiences a force proportional to the current.
- The result is a force proportional to and opposing the velocity.
- The dissipated energy shows up as I^2R heating of the cup.

Eddy-Current Damper

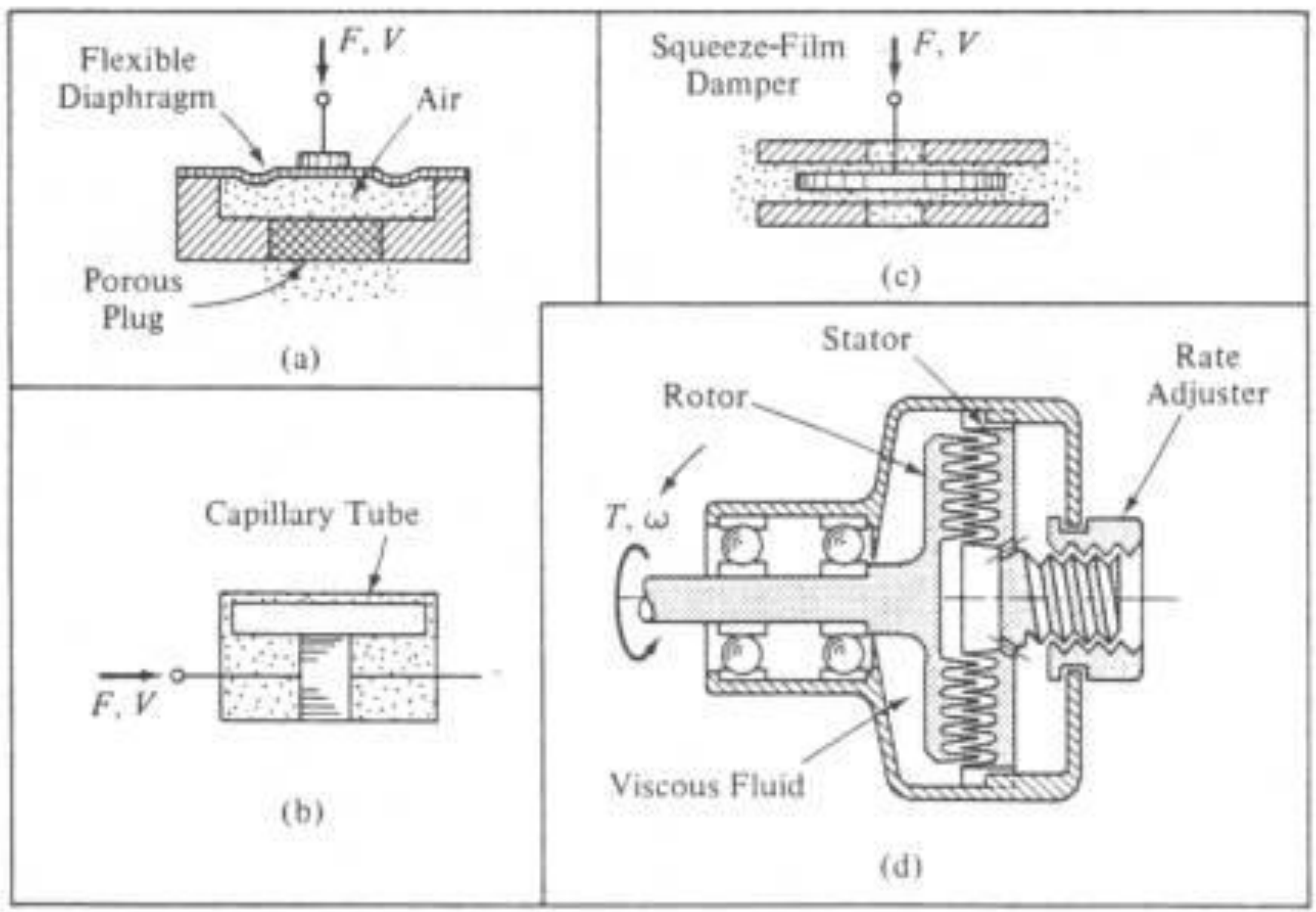


Cross-Section of Circular Configuration

Temperature Sensitivity of Damping Methods

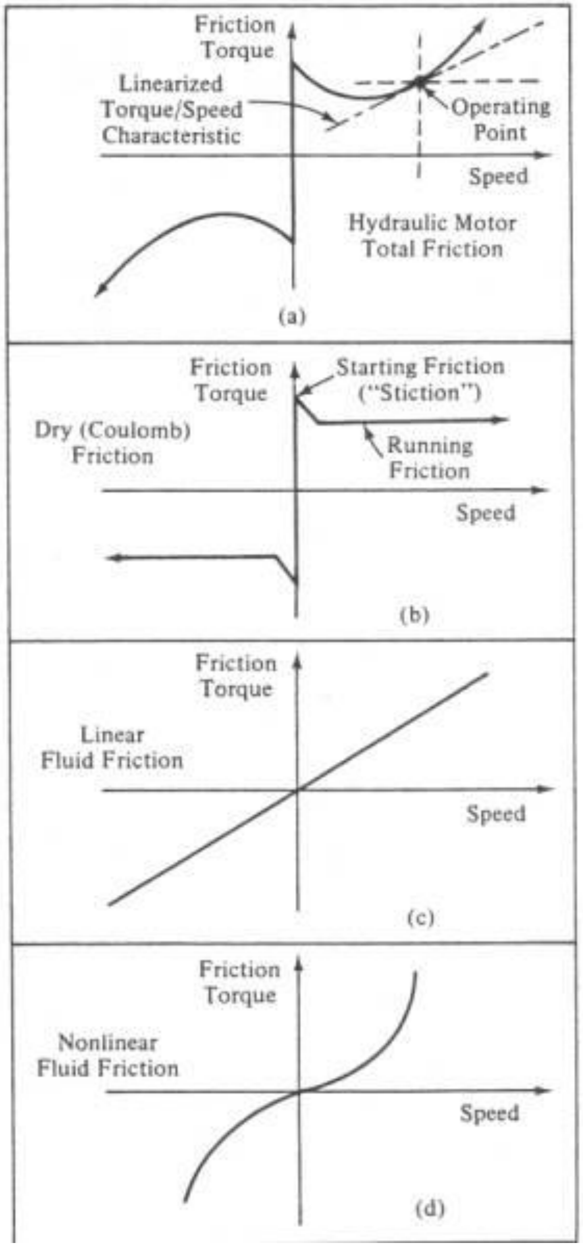


Other Examples of Damper Forms



- The damper element can also be used to represent unavoidable *parasitic* energy dissipation effects in mechanical systems.
  - Frictional effects in moving parts of machines
  - Fluid drag on vehicles (cars, ships, aircraft, etc.)
  - Windage losses of rotors in machines
  - Hysteresis losses associated with cyclic stresses in materials
  - Structural damping due to riveted joints, welds, etc.
  - Air damping of vibrating structural shapes

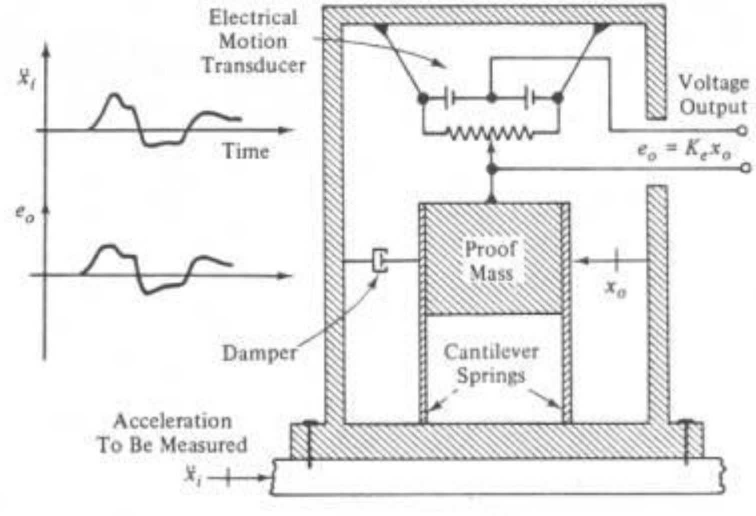
Hydraulic Motor Friction and its Components



Inertia Element

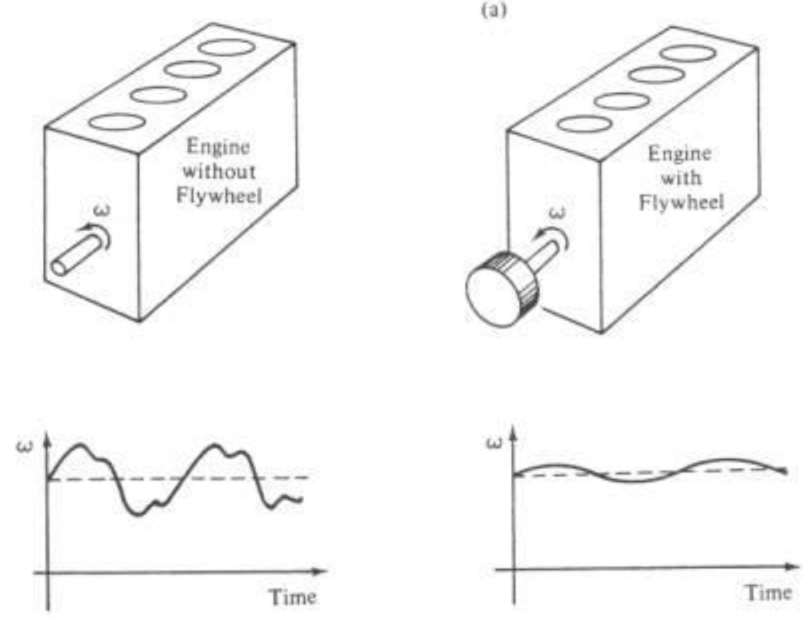
- A designer rarely inserts a component for the purpose of adding inertia; the mass or inertia element often represents an undesirable effect which is unavoidable since all materials have mass.
- There are some applications in which mass itself serves a useful function, e.g., accelerometers and flywheels.

Accelerometer



(a)

Useful Applications of Inertia



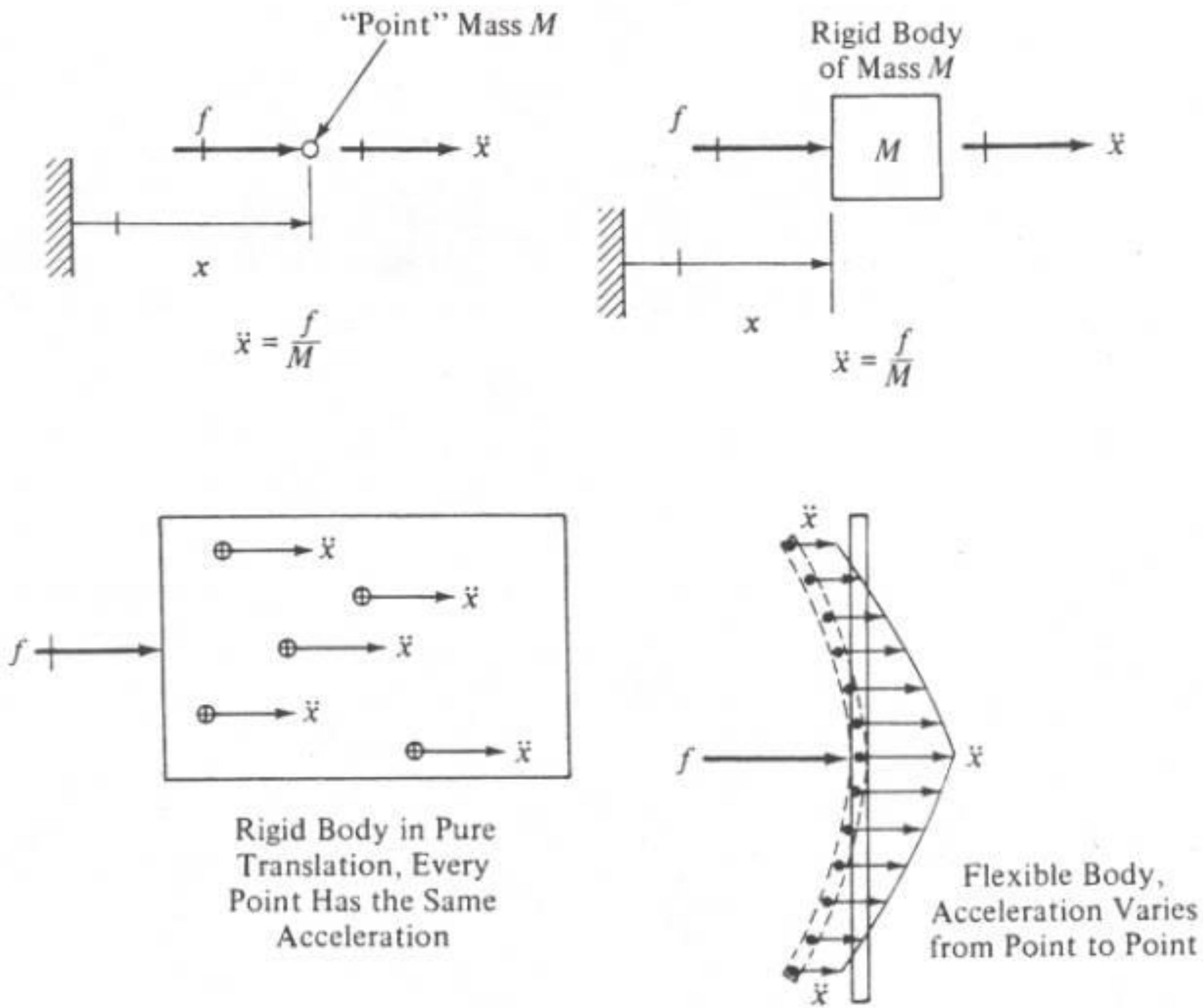
(b)

Flywheels are used as energy-storage devices or as a means of smoothing out speed fluctuations in engines or other machines.

- Newton's Law defines the behavior of mass elements and refers basically to an idealized "point mass":

$$\sum \text{forces} = (\text{mass})(\text{acceleration})$$

- The concept of rigid body is introduced to deal with practical situations. For pure translatory motion, every point in a rigid body has identical motion.
- Real physical bodies never display ideal rigid behavior when being accelerated.
- The pure / ideal inertia element is a model, not a real object.



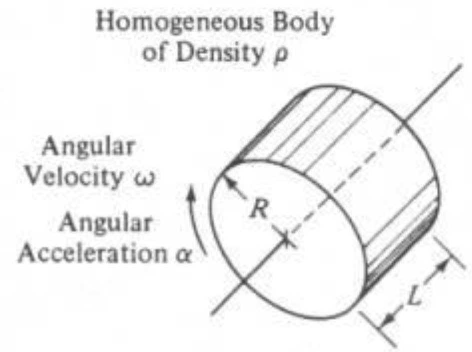
Rigid and Flexible Bodies: Definitions and Behavior

- Newton's Law in rotational form for bodies undergoing pure rotational motion about a single fixed axis:

$$\sum \text{torques} = (\text{moment of inertia})(\text{angular acceleration})$$

- The concept of moment of inertia J also considers the rotating body to be perfectly rigid.
- Note that to completely describe the inertial properties of any rigid body requires the specification of:
  - Its total mass
  - Location of the center of mass
  - 3 moments of inertia and 3 products of inertia

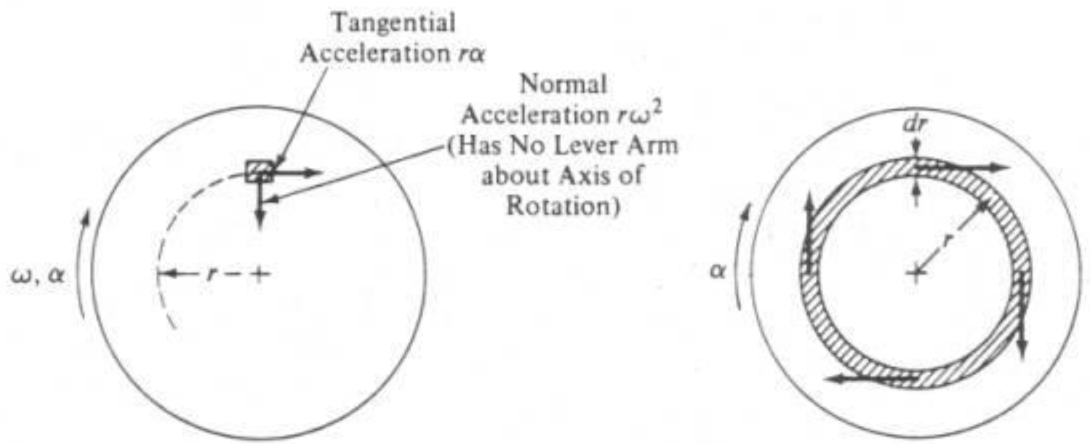
Rotational Inertia
J (kg-m^2)



tangential force

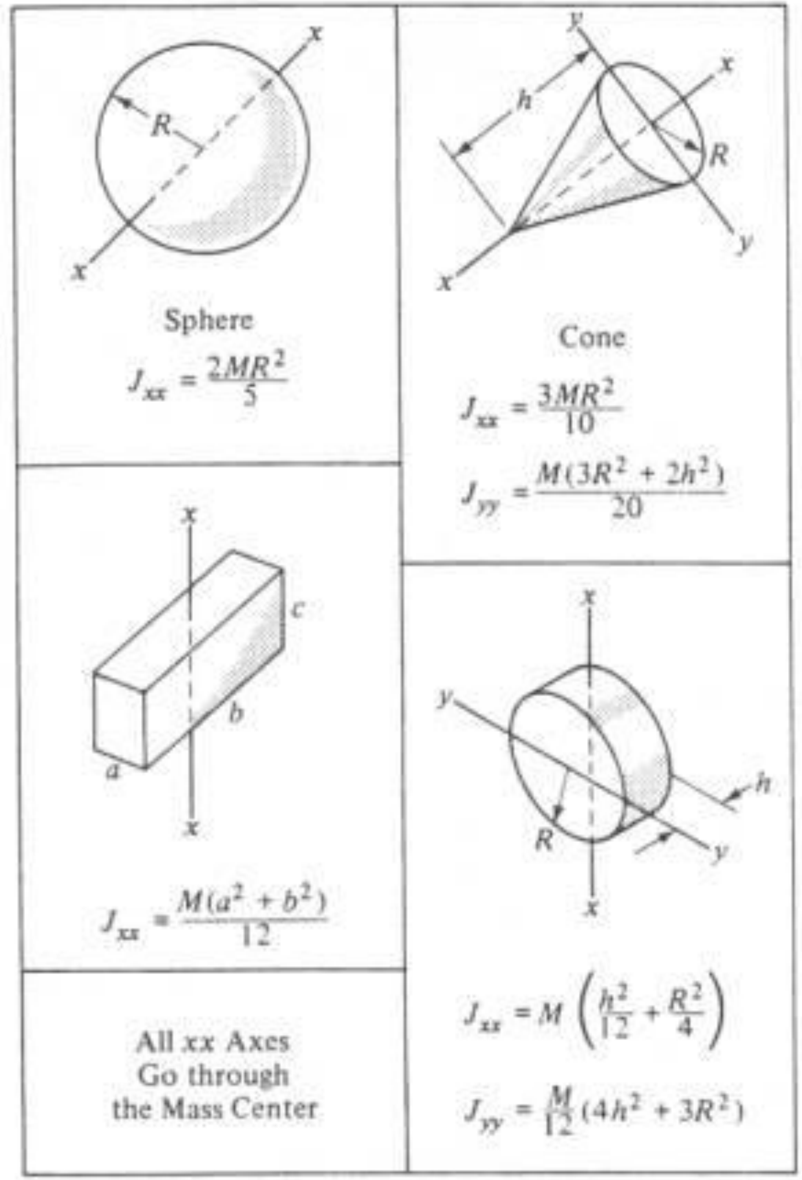
$$= (\text{mass})(\text{acceleration})$$

$$= \left[(2\pi r L \rho)(dr)\right](r\alpha)$$



$$\text{total torque} = \int_0^R \left(2\pi\rho L \alpha r^3\right) dr = \pi R^2 L \rho \frac{R^2}{2} = \frac{MR^2}{2}\alpha = J\alpha$$

Moments of Inertia for Some Common Shapes



- How do we determine J for complex shapes with possibly different materials involved?
  - In the design stage, where the actual part exists only on paper, estimate as well as possible!
  - Once a part has been constructed, use experimental methods for measuring inertial properties. How?

Experimental Measurement of Moment of Inertia

$$\sum(\text{torques}) = J\alpha = J\frac{d^2\theta}{dt^2}$$

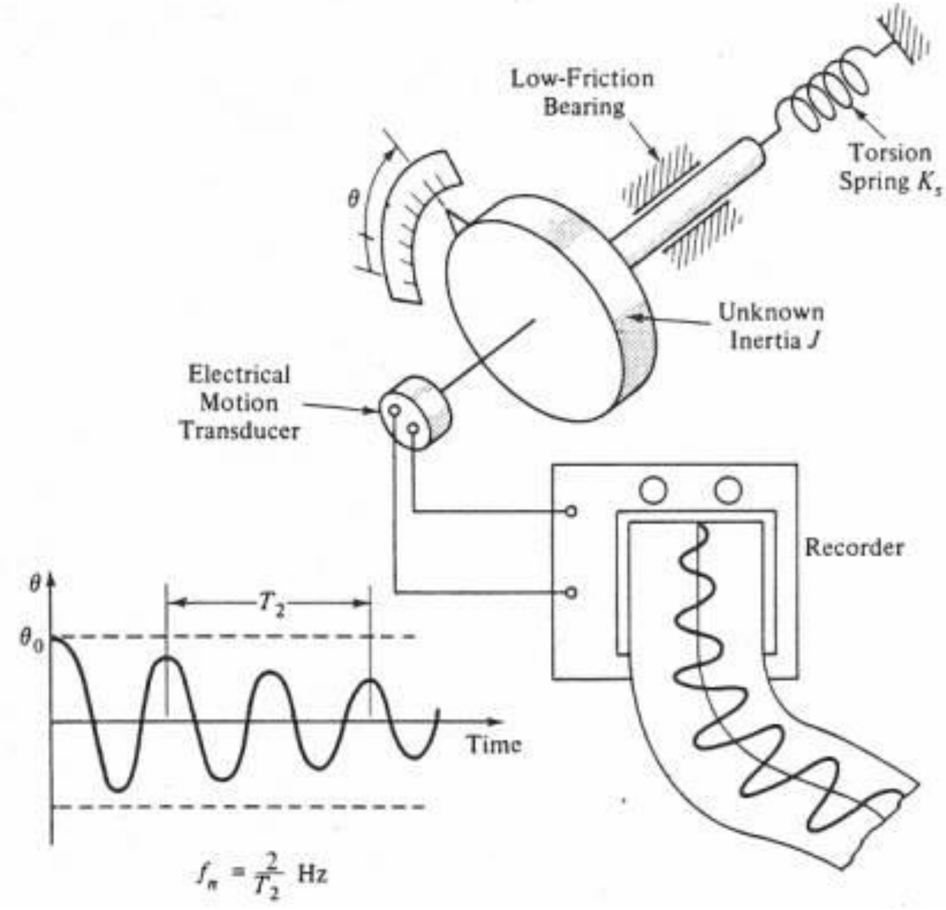
$$-K_s\theta = J\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{K_s}{J}\theta = 0$$

$$\theta = \theta_0 \cos\omega_n t \qquad (\dot{\theta}_0 = 0)$$

$$\omega_n = \sqrt{\frac{K_s}{J}} \quad \text{rad/sec}$$

$$f_n = \frac{\omega_n}{2\pi} \quad \text{cycles/sec}$$



$$J = \frac{K_s}{4\pi^2 f_n^{\,2}}$$

- Actually the oscillation will gradually die out due to the bearing friction not being zero.
- If bearing friction were pure Coulomb friction, it can be shown that the decay envelope of the oscillations is a straight line and that friction has no effect on the frequency.
- If the friction is purely viscous, then the decay envelope is an exponential curve, and the frequency of oscillation does depend on the friction but the dependence is usually negligible for the low values of friction in typical apparatus.

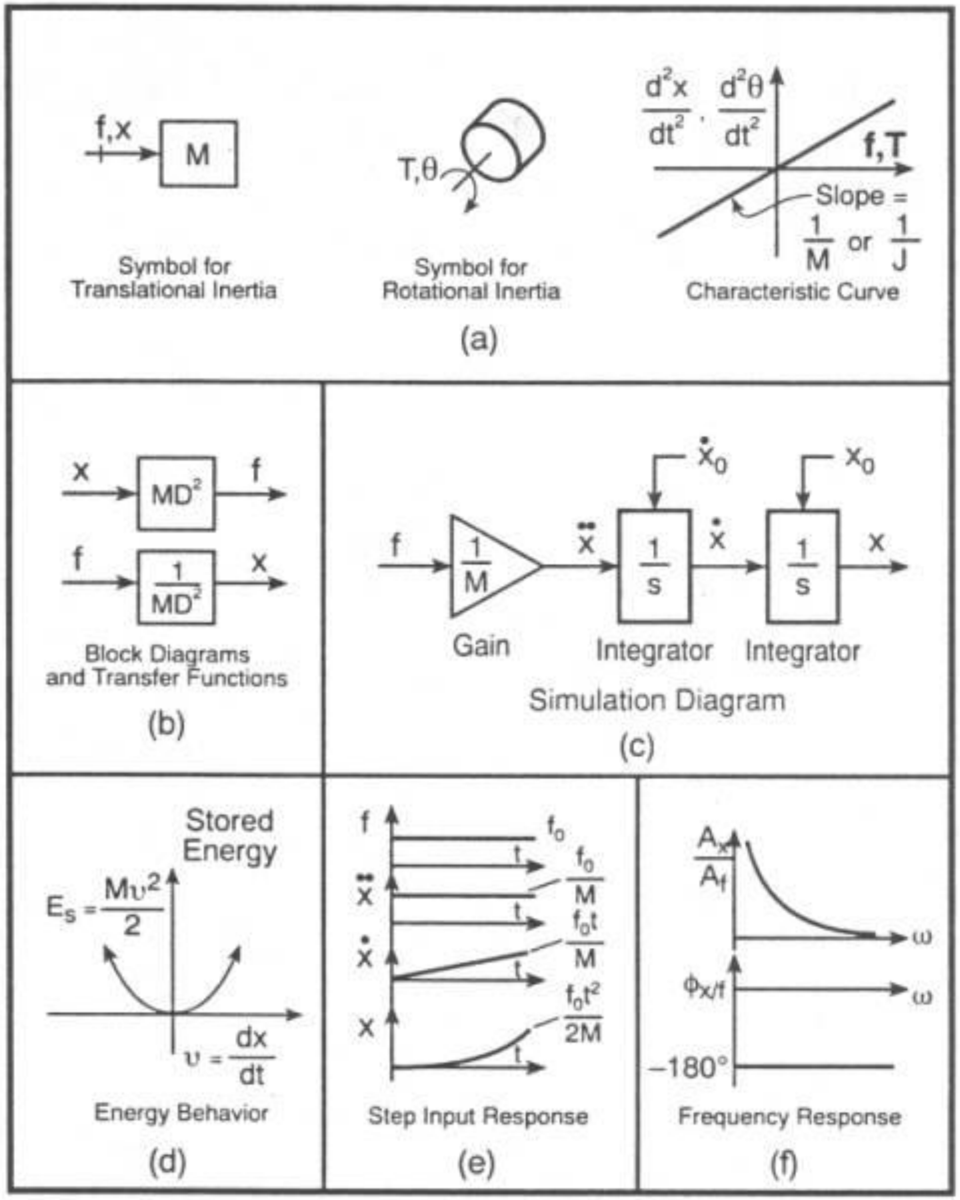
Inertia Element

Real inertias may be impure (have some springiness and friction) but are very close to ideal.

$$\frac{x}{f}(D) = \frac{1}{MD^2} \qquad \frac{\theta}{T}(D) = \frac{1}{JD^2}$$

Inertia Element stores energy as kinetic energy:

$$\frac{Mv^2}{2} \quad \text{or} \quad \frac{J\omega^2}{2}$$

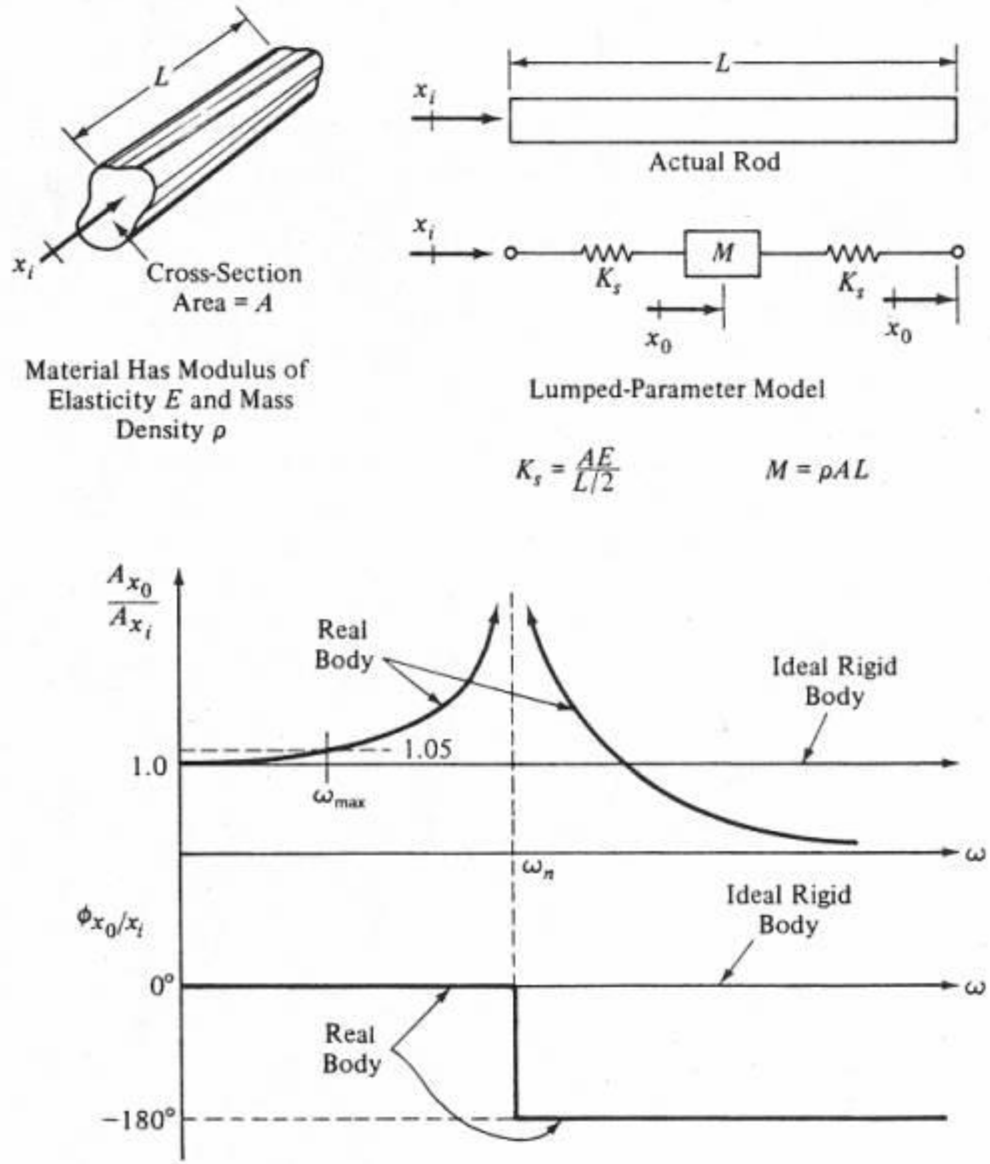


- A step input force applied to a mass initially at rest causes an instantaneous jump in acceleration, a ramp change in velocity, and a parabolic change in position.
- The frequency response of the inertia element is obtained from the sinusoidal transfer function:

$$\frac{x}{f}(i\omega) = \frac{1}{M(i\omega)^2} = \frac{1}{M\omega^2}\angle -180^\circ$$

  - At high frequency, the inertia element becomes very difficult to move.
  - The phase angle shows that the displacement is in a direction opposite to the applied force.

Useful Frequency Range for Rigid Model of a Real Flexible Body



A real flexible body approaches the behavior of a rigid body if the forcing frequency is small compared to the body's natural frequency.

Analysis

$$\left(x_i - x_o\right)\frac{2AE}{L} = \rho A L \ddot{x}_o$$

$$\frac{\rho L^2}{2E}\ddot{x}_o + x_o = x_i$$

$$\left(\frac{D^2}{\omega_n^2} + 1\right)x_o = x_i \qquad \omega_n = \sqrt{\frac{2E}{\rho L^2}}$$

$$\frac{x_i}{x_o}(D) = \frac{1}{\frac{D^2}{\omega_n^2} + 1} \qquad \frac{x_i}{x_o}(i\omega) = \frac{1}{\left(\frac{i\omega}{\omega_n}\right)^2 + 1} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

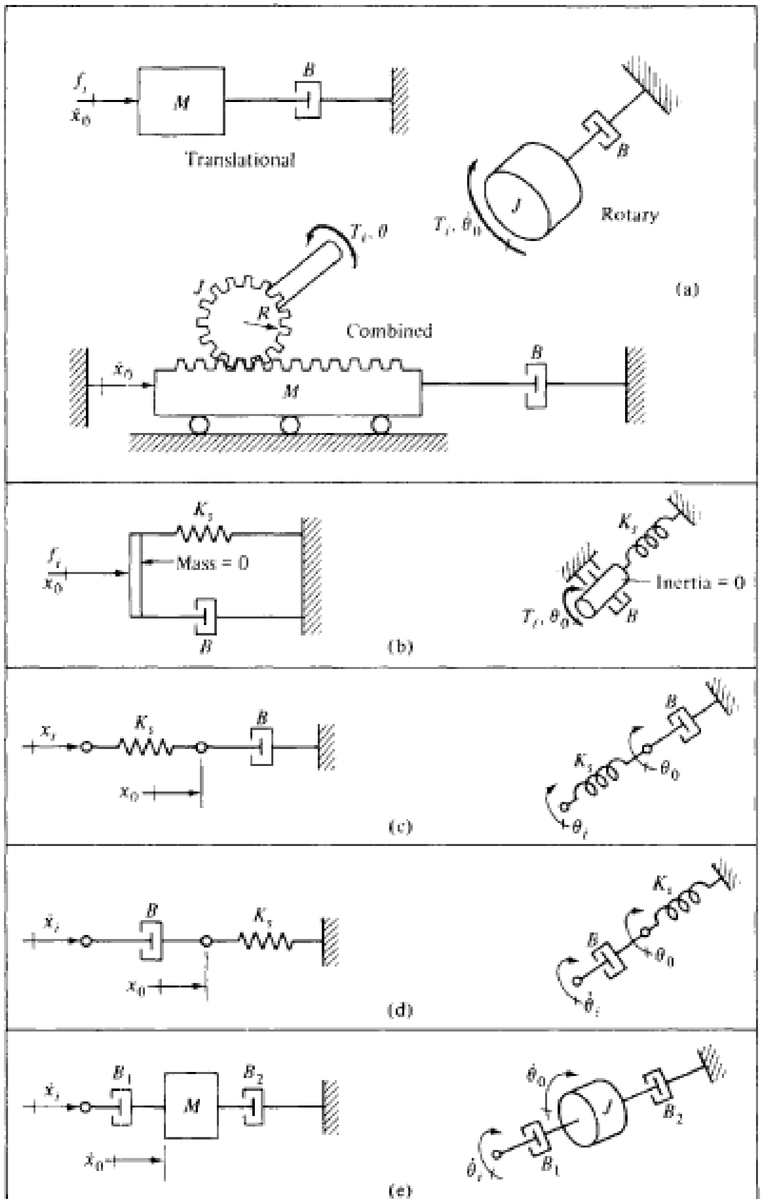
$$\frac{x_o}{x_i}(i\omega) = 1.05 = \frac{1}{1-\left(\frac{\omega_{max}}{\omega_n}\right)^2}$$

$$\omega_{max} = 0.218\omega_n = \frac{0.308}{L}\sqrt{\frac{E}{\rho}} \Rightarrow$$

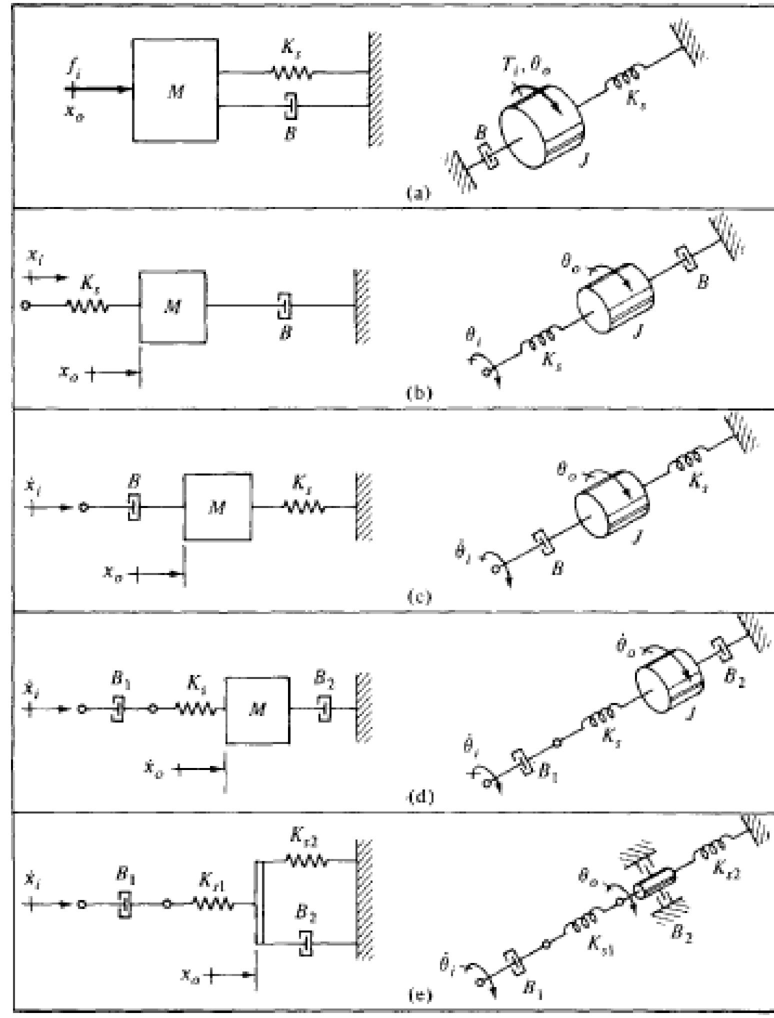
96200 cycles/min for a 6-inch steel rod

– ω_{max} is the highest frequency for which the real body behaves almost like an ideal rigid body.

- Frequency response is unmatched as a technique for defining the useful range of application for all kinds of dynamic systems.



1st-Order Mechanical Systems



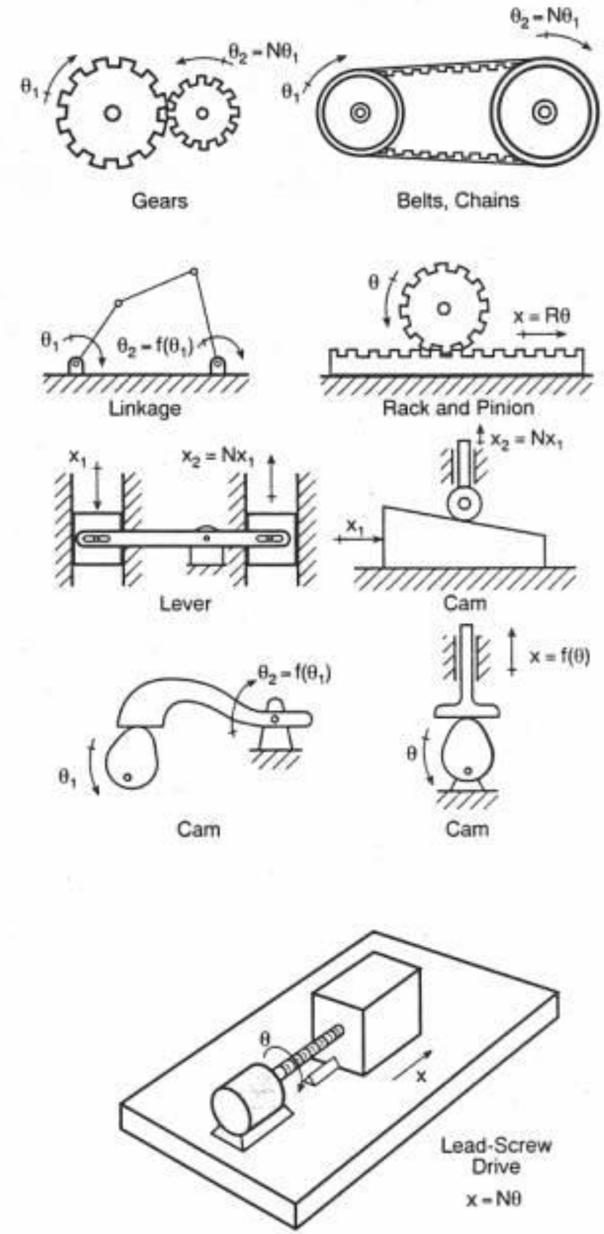
2nd-Order Mechanical Systems

Motion Transformers

- Mechanical systems often include mechanisms such as levers, gears, linkages, cams, chains, and belts.
- They all serve a common basic function, the transformation of the motion of an input member into the kinematically-related motion of an output member.
- The actual system may be simplified in many cases to a fictitious but dynamically equivalent one.

- This is accomplished by “referring” all the elements (masses, springs, dampers) and driving inputs to a single location, which could be the input, the output, or some selected interior point of the system.
- A single equation can then be written for this equivalent system, rather than having to write several equations for the actual system.
- This process is not necessary, but often speeds the work and reduces errors.

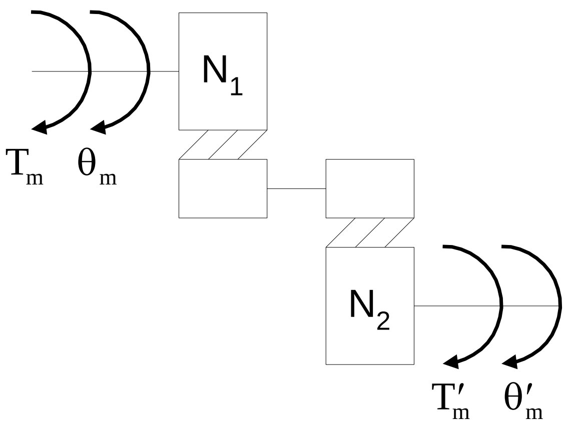
Motion Transformers



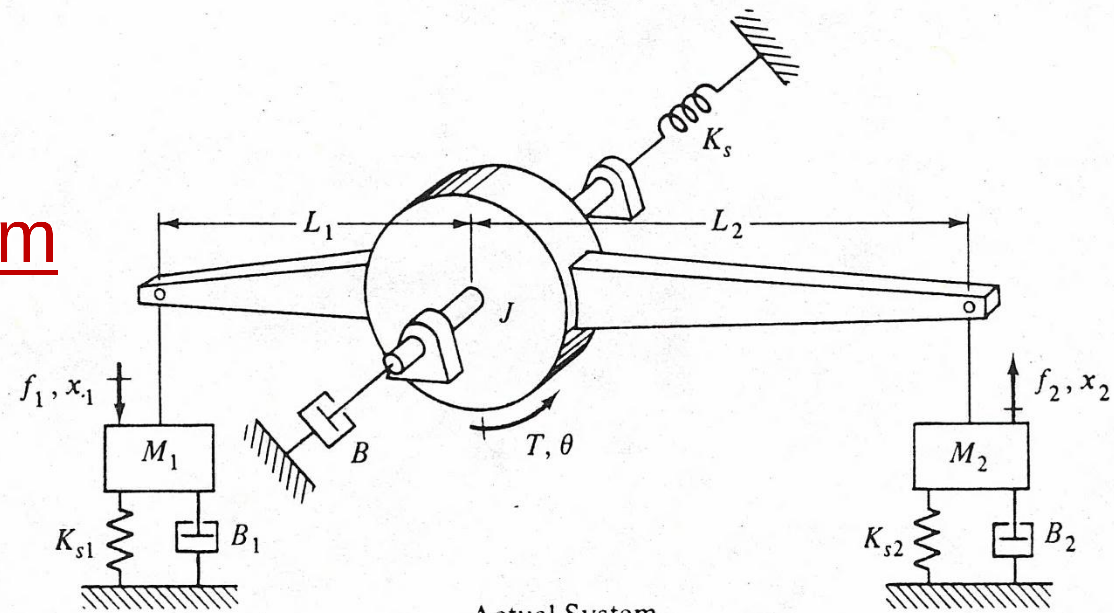
Gear Train Relations:

$$\frac{\theta_m}{\theta'_m} = \frac{N_2}{N_1} \equiv N$$

$$\frac{T_m}{T'_m} = \frac{N_1}{N_2} \equiv \frac{1}{N}$$



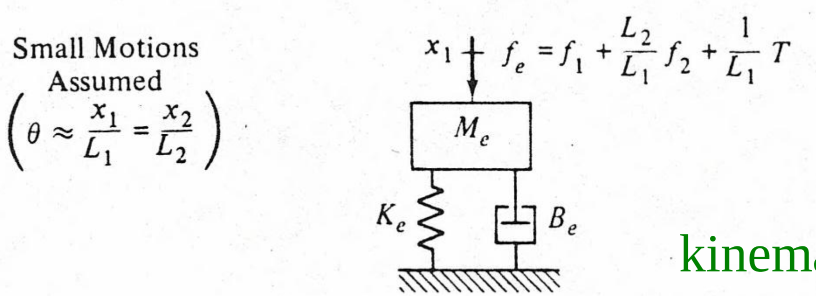
Translational Equivalent for A Complex System



Actual System

(a)

Refer all elements and inputs to the x_1 location and define a fictitious equivalent system whose motion will be the same as x_1 but will include all the effects in the original system.



Equivalent System Referred to x_1

(b)

x_1, x_2, θ are kinematically related

- Define a single equivalent spring element which will have the same effect as the three actual springs.
- Mentally apply a static force f_1 at location x_1 and write a torque balance equation:

$$f_1 L_1 = \left(K_{s1} x_1\right) L_1 + \left(\frac{L_2}{L_1} x_1 K_{s2}\right) L_2 + \frac{x_1 K_s}{L_1}$$

$$f_1 = K_{se} x_1$$

$$K_{se} = \left[K_{s1} + \left(\frac{L_2}{L_1}\right)^2 K_{s2} + \frac{1}{L_1^2} K_s\right]$$

– The equivalent spring constant K_{se} refers to a fictitious spring which, if installed at location x_1, would have exactly the same effect as all the springs together in the actual system.

– To find the equivalent damper, mentally remove the inertias and springs and again apply a force f_1 at x_1:

$$f_1 L_1 = \left(\dot{x}_1 B_1\right) L_1 + \left(\dot{x}_2 B_2\right) L_2 + B\dot{\theta}$$

$$= \dot{x}_1 B_1 L_1 + \frac{L_2^2}{L_1}\dot{x}_1 B_2 + \frac{\dot{x}_1}{L_1} B$$

$$f_1 = B_e \dot{x}_1$$

$$B_e = \left[B_1 + \left(\frac{L_2}{L_1}\right)^2 B_2 + \frac{1}{L_1^2} B \right]$$

– Finally, consider only the inertias present.

$$f_1 L_1 \approx \left(M_1 L_1^2\right)\frac{\ddot{x}_1}{L_1} + \left(M_2 L_2^2\right)\frac{\ddot{x}_1}{L_1} + \left(J\right)\frac{\ddot{x}_1}{L_1}$$

$$f_1 \approx M_e \ddot{x}_1$$

$$M_e = \left[M_1 + \left(\frac{L_2}{L_1}\right)^2 M_2 + \frac{1}{L_1^2} J\right]$$

– While the definitions of equivalent spring and damping constants are approximate due to the assumption of small motions, the equivalent mass has an additional assumption which may be less accurate; we have treated the masses as point masses, i.e., $J = ML^2$.

- To refer the driving inputs to the x_1 location we note that a torque T is equivalent to a force T/L_1 at the x_1 location, and a force f_2 is equivalent to a force $(L_2/L_1)f_2$.
- If we set up the differential equation of motion for this system and solve for its unknown x_1, we are guaranteed that this solution will be identical to that for x_1 in the actual system.
- Once we have x_1, we can get x_2 and/or θ immediately since they are related to x_1 by simple proportions.

– Rules for calculating the equivalent elements without deriving them from scratch:

- When referring a translational element (spring, damper, mass) from location A to location B, where A's motion is N times B's, multiply the element's value by N^2. This is also true for rotational elements coupled by motion transformers such as gears, belts, and chains.
- When referring a rotational element to a translational location, multiply the rotational element by $1/R^2$, where the relation between translation x and rotation θ (in radians) is $x = R\,\theta$. For the reverse procedure (referring a translational element to a rotational location) multiply the translational element by R^2.

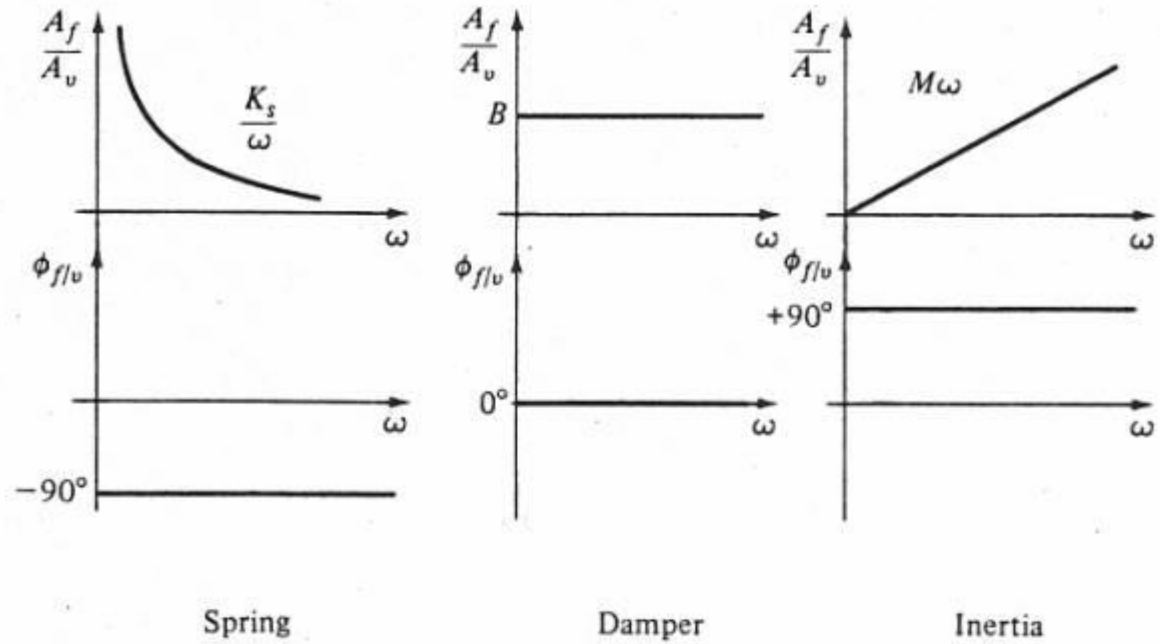
- When referring a force at A to get an equivalent force at B, multiply by N (holds for torques). Multiply a torque at θ by 1/R to refer it to x as a force. A force at x is multiplied by R to refer it as a torque to θ.

– These rules apply to any mechanism, no matter what its form, as long as the motions at the two locations are linearly related.

Mechanical Impedance

- When trying to predict the behavior of an assemblage of subsystems from their calculated or measured individual behavior, impedance methods have advantages.
- Mechanical impedance is defined as the transfer function (either operational or sinusoidal) in which force is the numerator and velocity the denominator. The inverse of impedance is called mobility.

Mechanical Impedance for the Basic Elements



$$Z_S(D) = \frac{f}{v}(D) = \frac{K_s}{D}$$

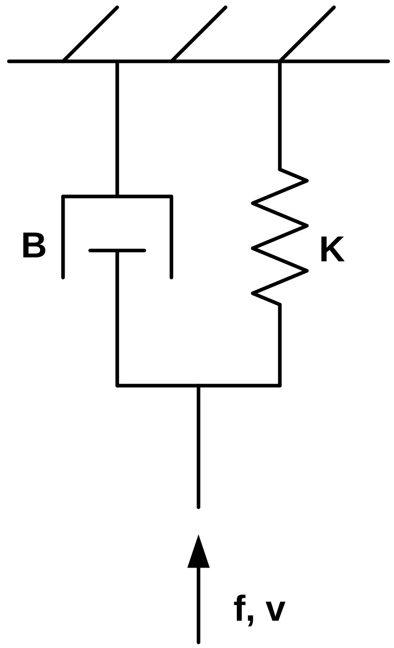
$$Z_B(D) = \frac{f}{v}(D) = B$$

$$Z_M(D) = \frac{f}{v}(D) = MD$$

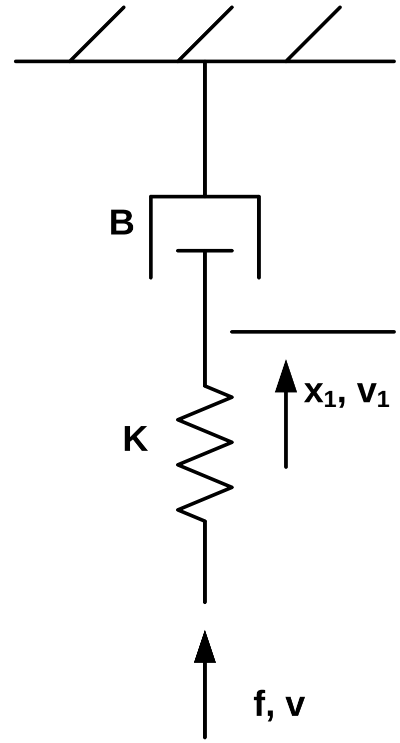
- Measurement of impedances of subsystems can be used to analytically predict the behavior of the complete system formed when the subsystems are connected. We can thus discover and correct potential design problems before the subsystems are actually connected.
- Impedance methods also provide "shortcut" analysis techniques.
  - When two elements carry the same force they are said to be connected in *parallel* and their combined impedance is the product of the individual impedances over their sum.

– For impedances which have the same velocity, we say they are connected in *series* and their combined impedance is the sum of the individual ones.

– Consider the following systems:



Series Connection



Parallel Connection

– Parallel Connection

$$\frac{f}{v}(D) = \frac{\frac{K}{D}B}{\frac{K}{D}+B} = \frac{KB}{BD+K}$$

– Series Connection

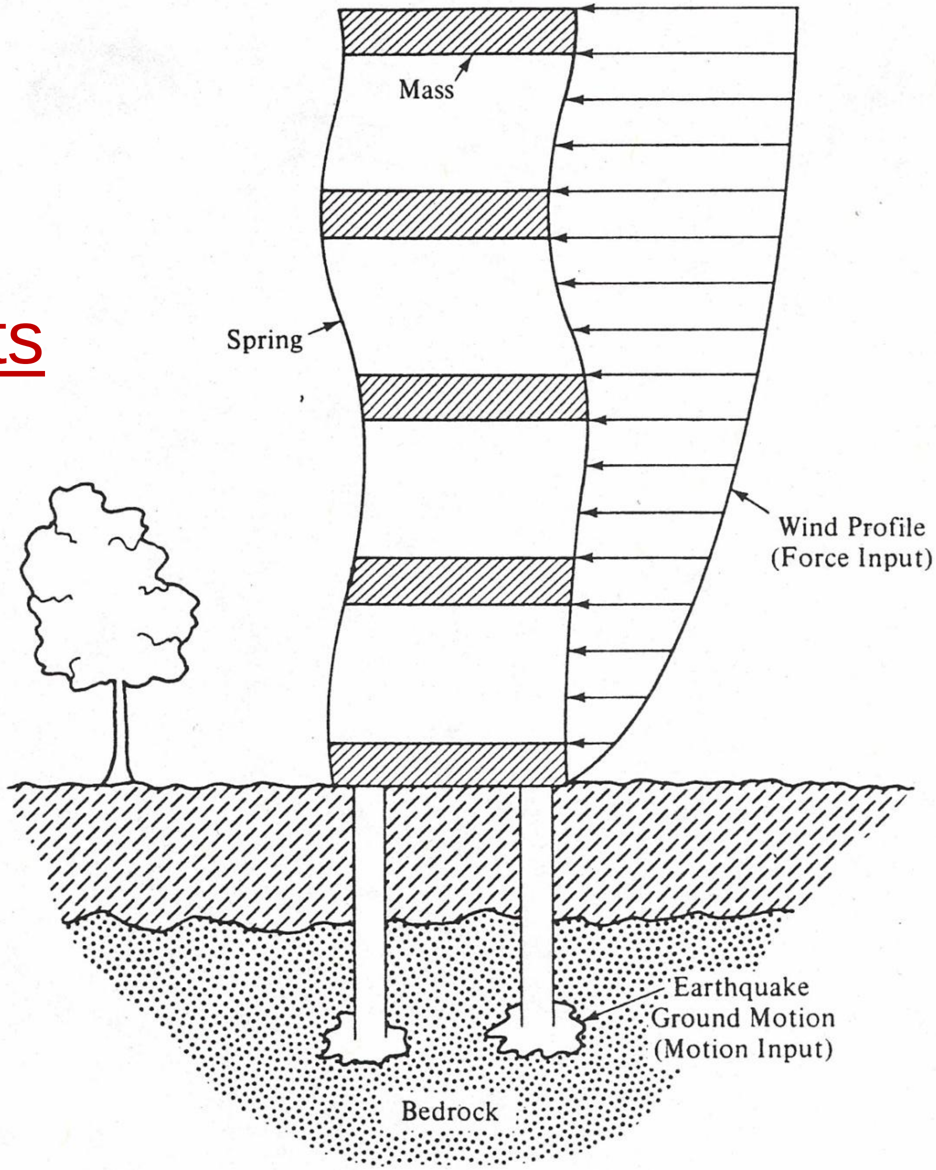
$$\frac{f}{v}(D) = B + \frac{K}{D} = \frac{BD+K}{D}$$

Force and Motion Sources

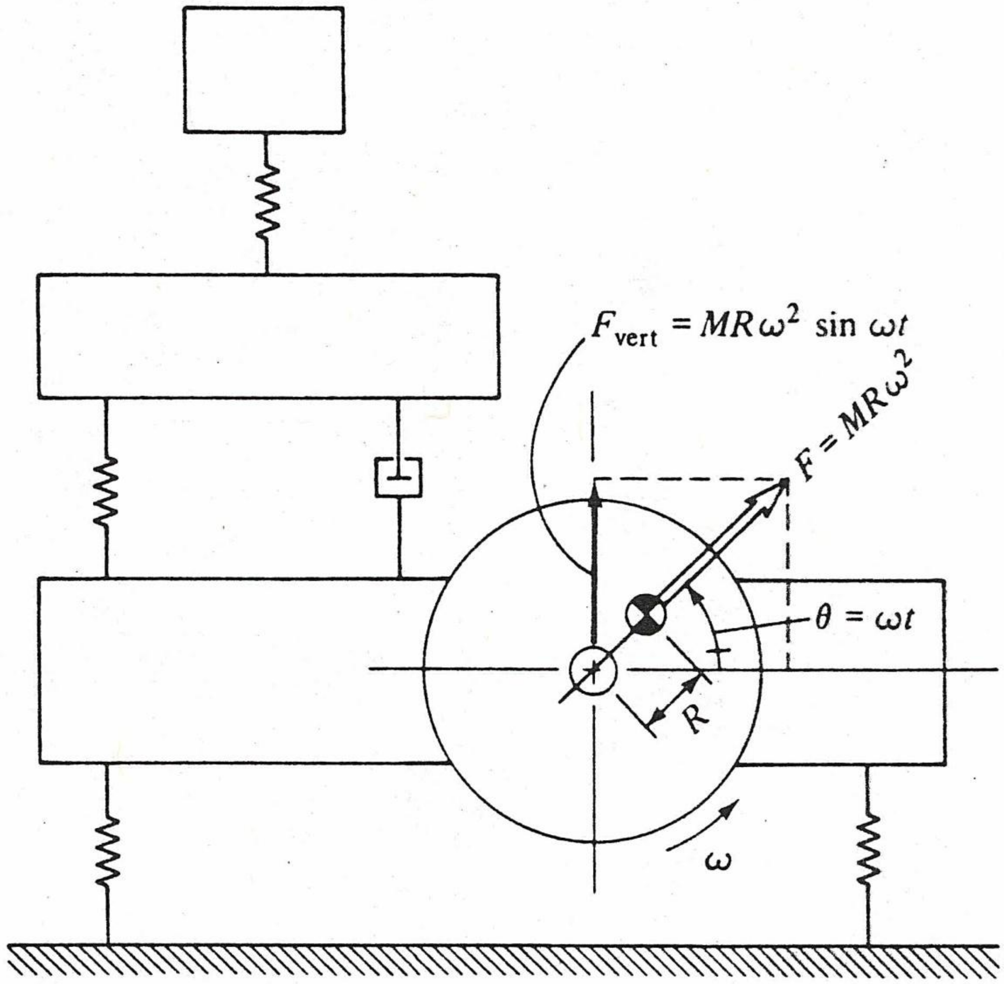
- The ultimate driving agency of any mechanical system is always a force not a motion; force causes acceleration, acceleration does not cause force.
- Motion does not occur without a force occurring first.
- At the input of a system, what is known, force or motion? If motion is known, then this motion was caused by some (perhaps unknown) force and postulating a problem with a motion input is acceptable.

- There are only <u>two classes of forces</u>:
  - Forces associated with physical contact between two bodies
  - Action-at-a-distance forces, i.e., gravitational, magnetic, and electrostatic forces
- There are no other kinds of forces! (Inertia force is a fictitious force.)
- The choice of an input form to be applied to a system requires careful consideration, just as the choice of a suitable model to represent a component or system.
- Here are some examples of force and motion sources.

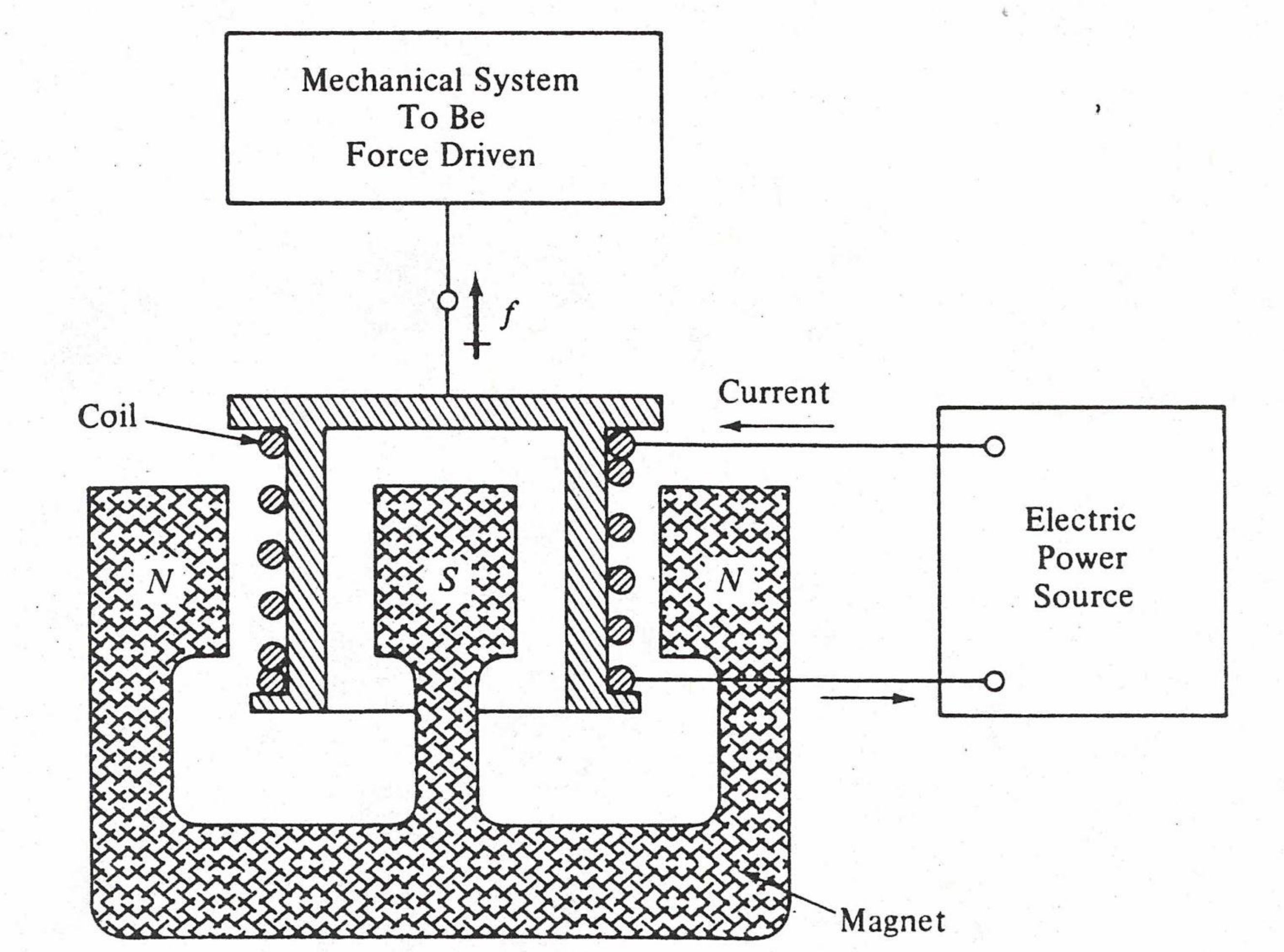
Force and Motion Inputs acting on a Multistory Building



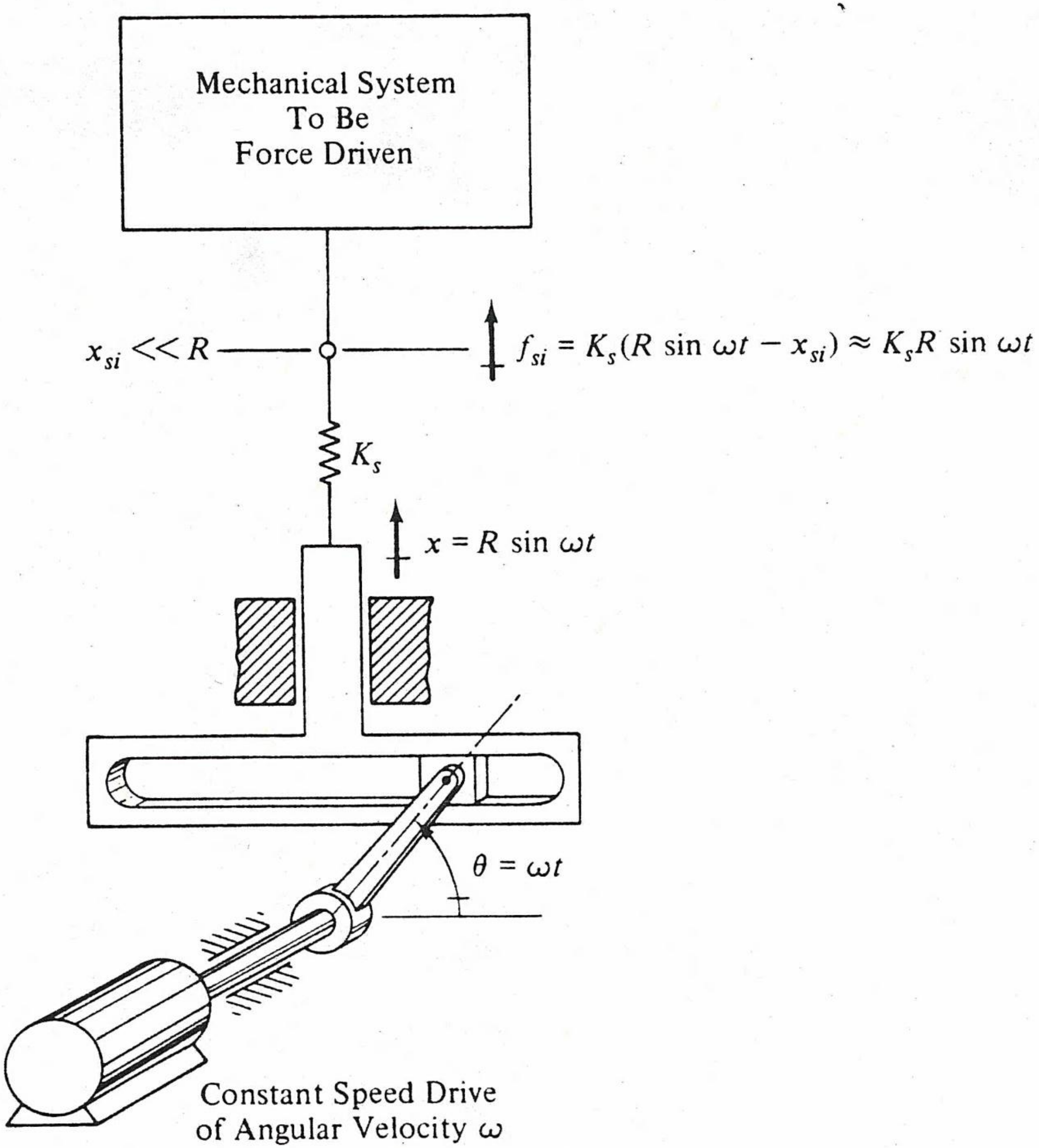
A Mechanical Vibration Shaker: Rotating Unbalance as a Force Input



Electrodynamic Vibration Shaker as a Force Source



Force Source Constructed from a Motion Source and a Soft Spring



- Energy Considerations
  - A system can be caused to respond only by the source supplying some energy to it; an interchange of energy must occur between source and system.
  - If we postulate a force source, there will be an associated motion occurring at the force input point.
  - The instantaneous power being transmitted through this energy port is the product of instantaneous force and velocity.
  - If the force applied by the source and the velocity caused by it are in the same direction, power is supplied by the source to the system. If force and velocity are opposed, the system is returning power to the source.

– The concept of mechanical impedance is of some help here.

– The transfer function relating force and velocity at the input port of a system is called the <u>driving-point impedance</u> Z_{dp}.

$$Z_{dp}(D) = \frac{f}{v}(D)$$

$$Z_{dp}(i\omega) = \frac{f}{v}(i\omega)$$

– We can write an expression for power:

$$P = fv = f\frac{f}{Z_{dp}} = \frac{f^2}{Z_{dp}}$$

– If we apply a force source to a system with a high value of driving-point impedance, not much power will be taken from the source, since the force produces only a small velocity. The extreme case of this would the application of a force to a perfectly rigid wall (driving-point impedance is infinite, since no motion is produced no matter how large a force is applied). In this case the source would not supply any energy.
– The higher the driving-point impedance, the more a real force source behaves like an ideal force source.
– The lower the driving-point impedance, the more a real motion source behaves like an ideal motion source.

- Real sources may be described accurately as combinations of ideal sources and an output impedance characteristic of the physical device.
- A complete description of the situation thus requires knowledge of two impedances:
  - The output impedance of the real source
  - The driving-point impedance of the driven system