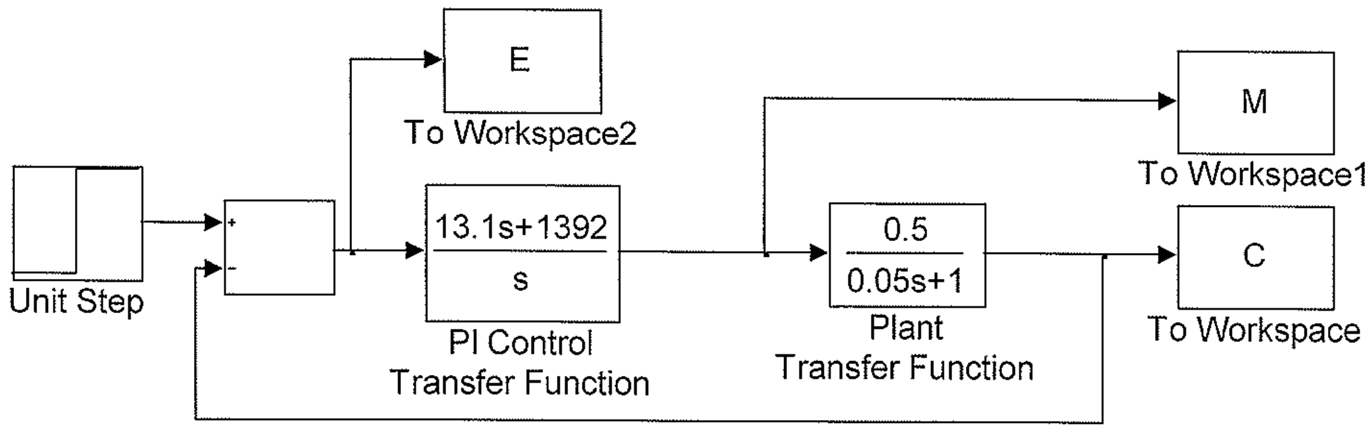
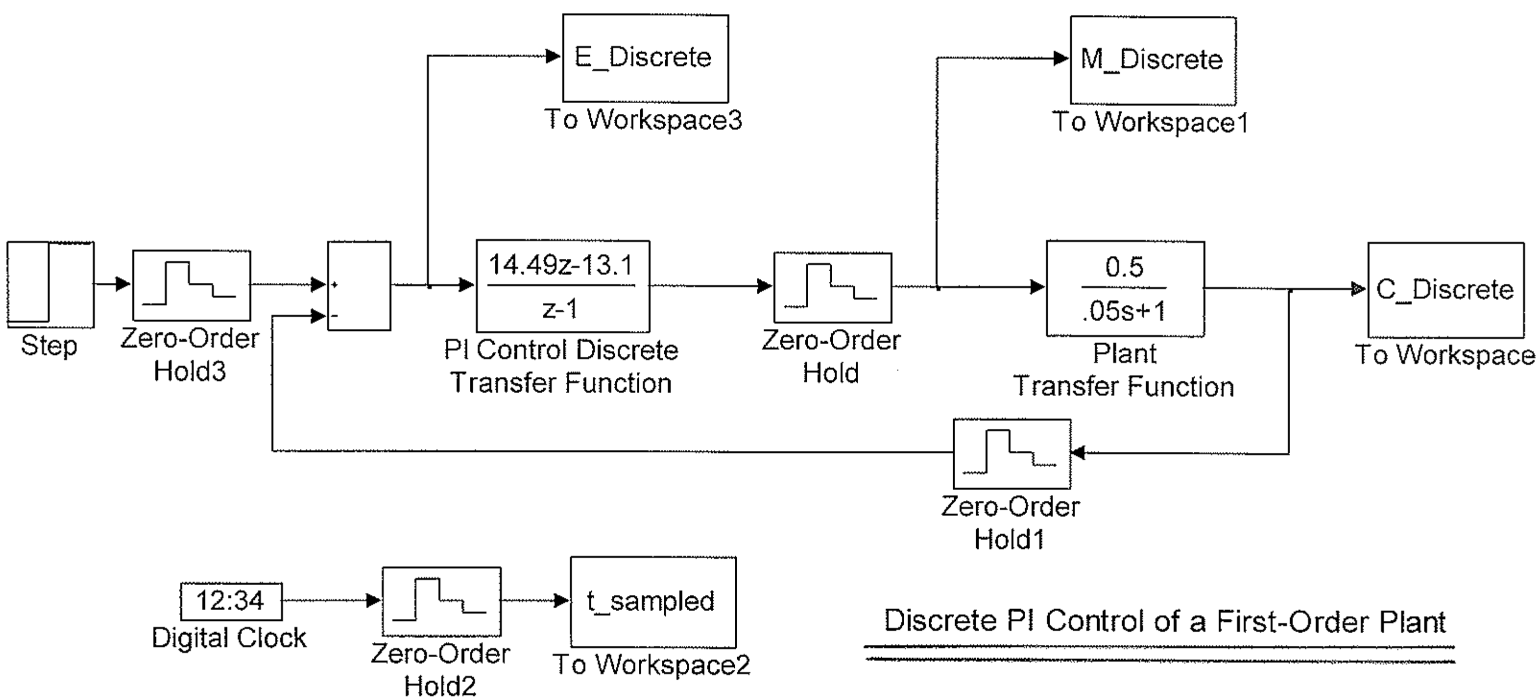
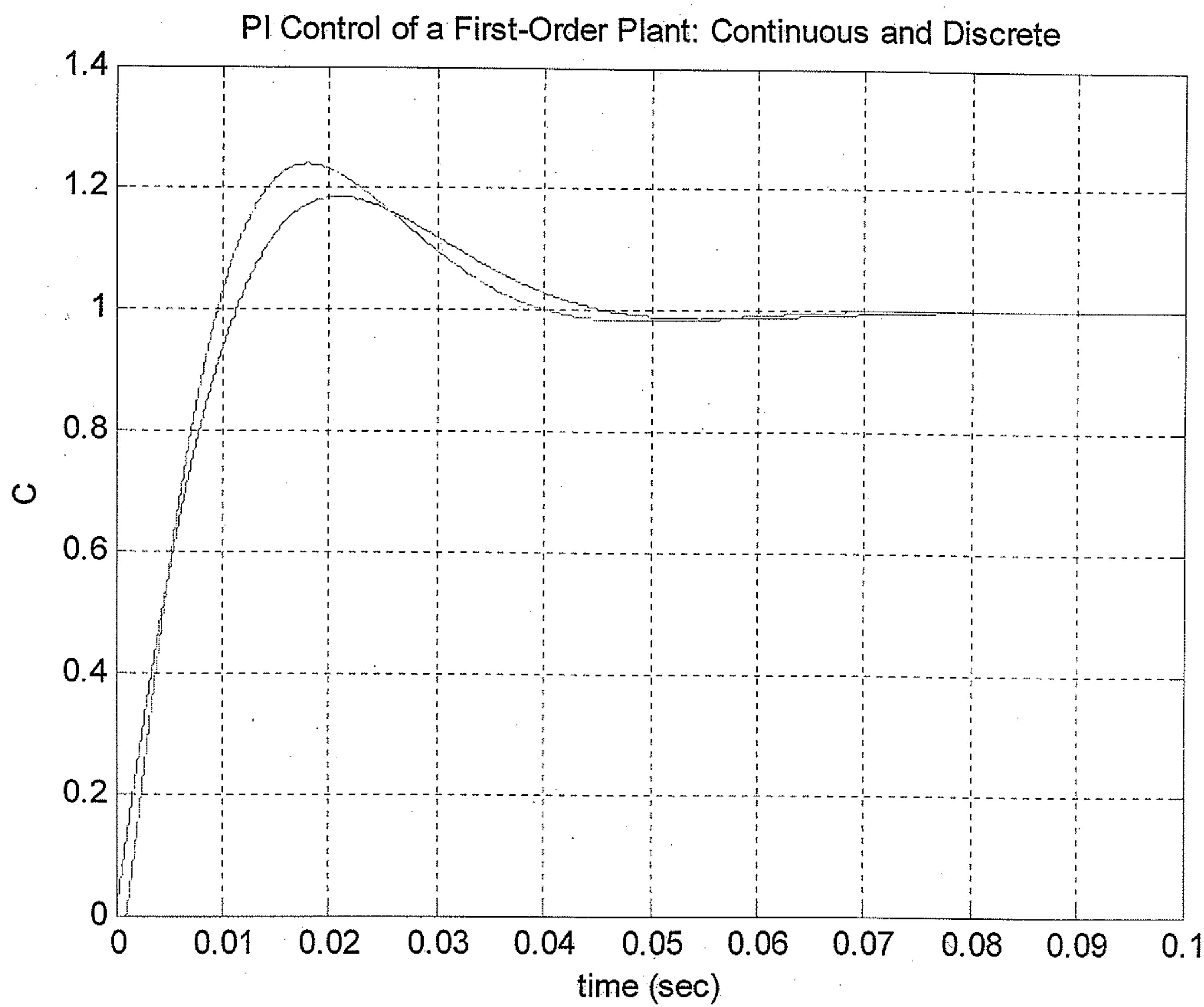
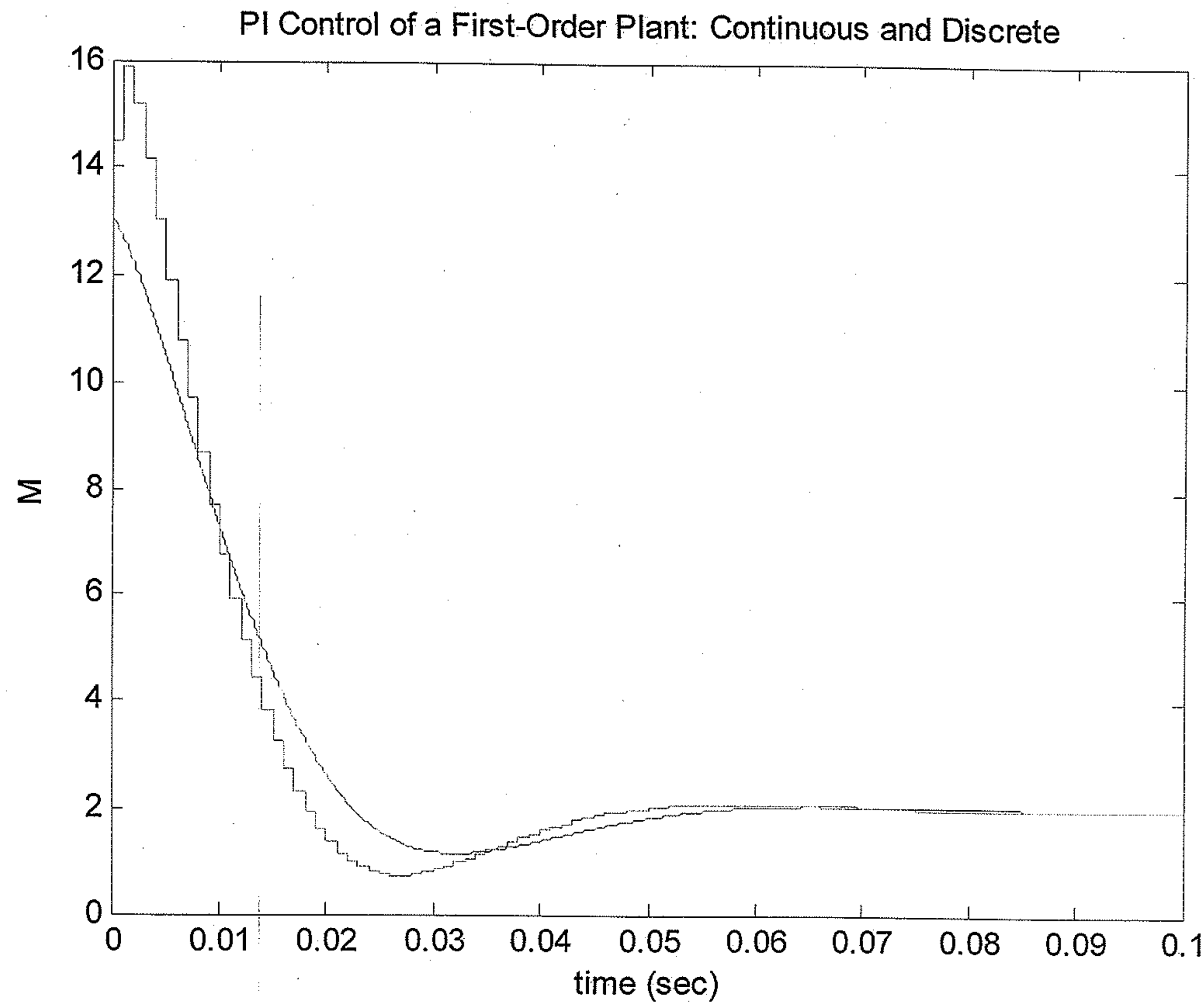


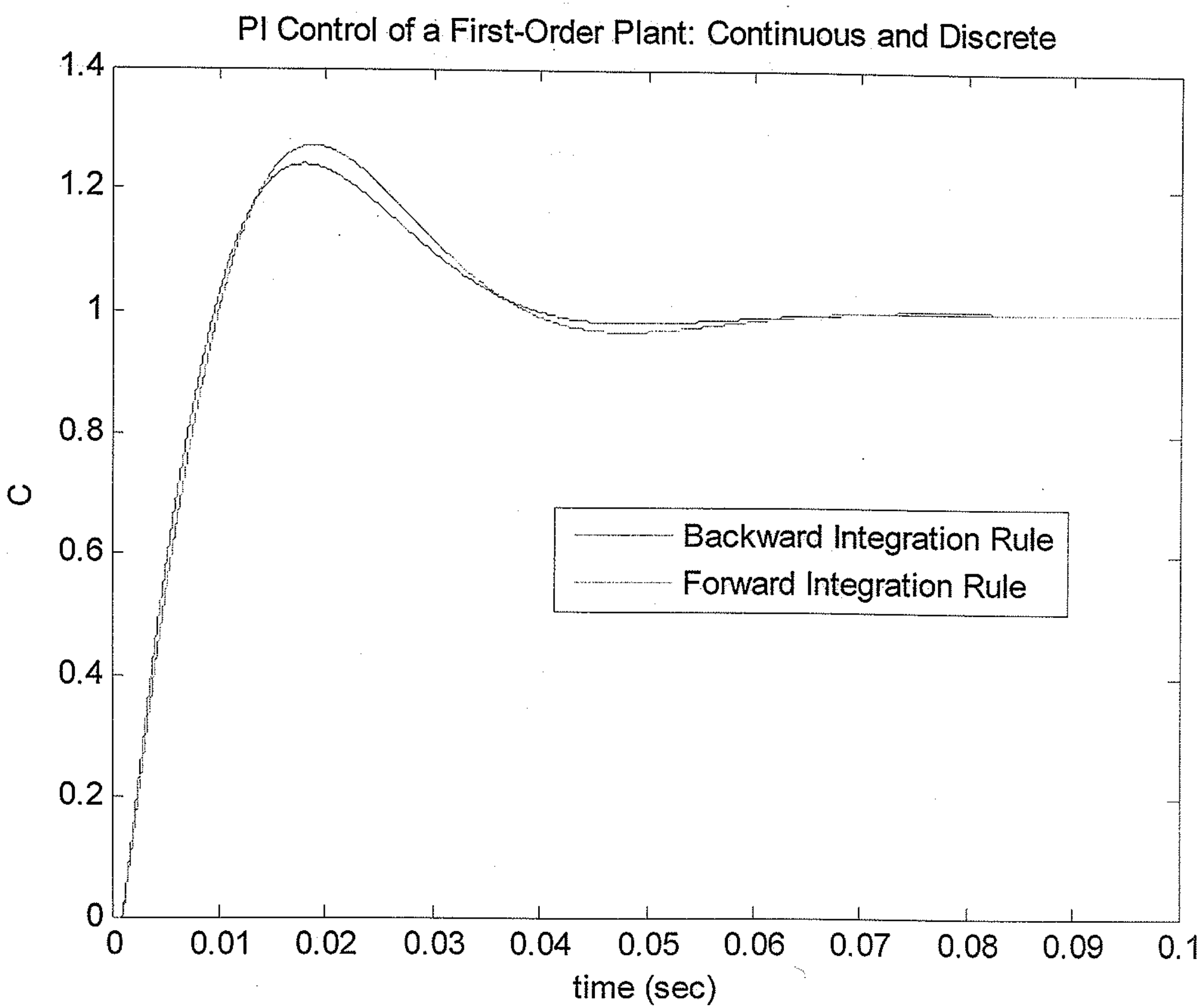
PI Control of a First-Order Plant



Discrete PI Control of a First-Order Plant


PI Control of a First-Order Plant: Continuous and Discrete
1.4
1.2
1
0.8
0.6
0.4
0.2
0
C
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
time (sec)


PI Control of a First-Order Plant: Continuous and Discrete
M
16
14
12
10
8
6
4
2
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
time (sec)


PI Control of a First-Order Plant: Continuous and Discrete
C
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
time (sec)
Backward Integration Rule
Forward Integration Rule

Computer Control PID

$$E \longrightarrow \boxed{PID} \longrightarrow M$$

Proportional Control

$$m(t) = K_p e(t)$$ Differential Equation

$$\frac{M}{E} = K_p$$ Transfer function - D operator

$$\left.\begin{aligned} m_n &= K_p e_n \\ m_{n-1} &= K_p e_{n-1} \end{aligned}\right\}$$

Difference Equation

$$m_n - m_{n-1} = K_p (e_n - e_{n-1})$$

$$m_n - B m_n = K_p (e_n - B e_n)$$ B = backshift operator

$$\begin{cases} B m_n = m_{n-1} \\ B^2 m_n = m_{n-2} \end{cases}$$

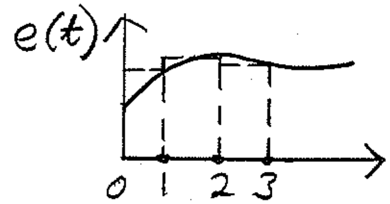
$$\left.\begin{aligned} (1-B) m_n &= K_p (1-B) e_n \\ \frac{m_n}{e_n} &= K_p \end{aligned}\right\}$$ discrete transfer functio

Integral Control

$$m(t) = K_I \int_0^t e(\tau)\,d\tau$$ Differential Equation

$$M = \frac{K_I}{D} E \Rightarrow \frac{M}{E} = \frac{K_I}{D}$$ Transfer function, D operator

$$m_n = K_I \sum_{j=1}^{n} T e_j$$

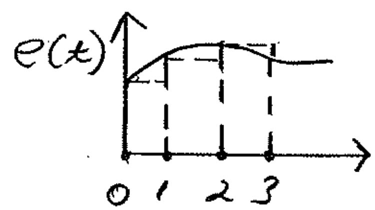


$n = \frac{t}{T}$

T = sample period

backward rectangle rule

$$m_n = K_I \sum_{j=0}^{n-1} T e_j$$



$n = \frac{t}{T}$

forward rectangle rule

Backward Rule $\Rightarrow$ $$m_n = K_I \sum_{j=1}^{n-1} T e_j + K_I T e_n = m_{n-1} + K_I T e_n$$

$$m_n - m_{n-1} = K_I T e_n$$

Forward Rule $\Rightarrow$ $$m_n = K_I \sum_{j=0}^{n-2} T e_j + K_I T e_{n-1} = m_{n-1} + K_I T e_{n-1}$$

$$m_n - m_{n-1} = K_I T e_{n-1}$$

Backward Rule

$$m_n - \beta m_n = K_I T e_n$$

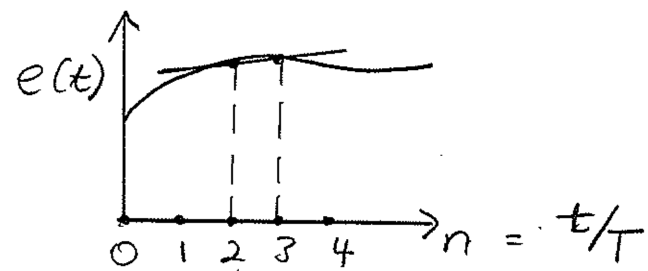
$$(1-\beta) m_n = K_I T e_n \qquad \frac{m_n}{e_n} = \frac{K_I T}{1-\beta}$$

Forward Rule

$$m_n - \beta m_n = K_I T \beta e_n$$

$$(1-\beta) m_n = K_I T \beta e_n \qquad \frac{m_n}{e_n} = \frac{K_I T \beta}{1-\beta}$$

Derivative Control



Backward Difference Rule

Diff. Eq. → $$m(t) = K_D \frac{de(t)}{dt}$$

$$M = K_D D E \qquad \frac{M}{E} = K_D D \qquad \text{Transfer Function}$$

$$\frac{de(t)}{dt} = \frac{e_n - e_{n-1}}{T}$$

$$m_n = K_D \left(\frac{e_n - e_{n-1}}{T} \right)$$

$$m_{n-1} = K_D \left(\frac{e_{n-1} - e_{n-2}}{T} \right)$$

Difference Equation

$$m_n - m_{n-1} = \frac{K_D}{T} \left(e_n - 2e_{n-1} + e_{n-2} \right)$$

$$(1-B)\, m_n = \frac{K_D}{T}(1 - 2B + B^2)\, e_n$$

$$\frac{m_n}{e_n} = \frac{K_D}{T}\left(\frac{1-2B+B^2}{1-B}\right)$$

Summary: PID Control

$m_n - m_{n-1} = K_P(e_n - e_{n-1})$ Proportional

$m_n - m_{n-1} = K_I T e_n$ Backward Rule

$m_n - m_{n-1} = K_I T e_{n-1}$ Forward Rule

$\Big\}$ Integral

$m_n - m_{n-1} = \frac{K_D}{T}(e_n - 2e_{n-1} + e_{n-2})$ Derivative

Control Algorithm

$$m_n - m_{n-1} = K_0 e_n + K_1 e_{n-1} + K_2 e_{n-2}$$

$$m_n - m_{n-1} = K_P e_n - K_P e_{n-1} + K_I T e_n + \frac{K_D}{T}(e_n - 2e_{n-1} + e_{n-2})$$

<u>Backward Rule</u>

$$= \left(K_P + K_I T + \frac{K_D}{T}\right) e_n + \left(-K_P - \frac{2K_D}{T}\right) e_{n-1} + \left(\frac{K_D}{T}\right) e_{n-2}$$

$$m_n - m_{n-1} = K_P e_n - K_P e_{n-1} + K_I T e_{n-1} + \frac{K_D}{T}(e_n - 2e_{n-1} + e_{n-2})$$

<u>Forward Rule</u>

$$= \left(K_P + \frac{K_D}{T}\right) e_n + \left(-K_P + K_I T - \frac{2K_D}{T}\right) e_{n-1} + \left(\frac{K_D}{T}\right) e_{n-2}$$

$$m_n - m_{n-1} = K_0 e_n + K_1 e_{n-1} + K_2 e_{n-2}$$

$$\left.\begin{aligned} K_0 &= K_p + K_I T + \frac{K_D}{T} \\ K_1 &= -K_p - \frac{2K_D}{T} \\ K_2 &= \frac{K_D}{T} \end{aligned}\right\} \text{with Backward Rule}$$

$$\left.\begin{aligned} K_0 &= K_p + \frac{K_D}{T} \\ K_1 &= -K_p + K_I T - \frac{2K_D}{T} \\ K_2 &= \frac{K_D}{T} \end{aligned}\right\} \text{with forward Rule}$$

$$(1-B)m_n = (K_0 + K_1 B + K_2 B^2)e_n \qquad B = 1/z$$

$$\frac{m_n}{e_n} = \frac{K_0 + K_1 B + K_2 B^2}{1-B} = \frac{K_0 z^2 + K_1 z + K_2}{z(1-z)}$$

PI Controller

$$\frac{m_n}{e_n} = \frac{K_0 + K_1 B}{1 - B} = \frac{K_0 z + K_1}{z - 1} \qquad B = 1/z$$

$$\left.\begin{aligned} K_0 &= K_P + K_I T \\ K_1 &= -K_P \end{aligned}\right\} \text{ Backward Rule}$$

$$\left.\begin{aligned} K_0 &= K_P \\ K_1 &= -K_P + K_I T \end{aligned}\right\} \text{ Forward Rule}$$

$$\frac{M}{E}(D) = K_P + \frac{K_I}{D} = \frac{K_P D + K_I}{D} = \frac{K_P s + K_I}{s}$$

$$\frac{M}{E}(B) = \frac{K_0 + K_1 B}{1 - B} = \frac{K_0 z + K_1}{z - 1}$$

with K_0 and K_1 defined above

Specific Case

$$K_p = 13.1 \qquad K_I = 1392 \qquad \frac{13.1\,s + 1392}{s}$$

$$T = .001$$

$$\left.\begin{aligned} K_0 &= K_p + K_I T = 14.49 \\ K_1 &= -K_p = -13.1 \end{aligned}\right\} \quad \frac{14.49\,z - 13.1}{z-1}$$

$$\left.\begin{aligned} K_0 &= K_p = 13.1 \\ K_1 &= -K_p + K_I T = -11.71 \end{aligned}\right\} \quad \frac{13.1\,z - 11.71}{z-1}$$

MatLab $\Longrightarrow$

```
control = tf([13.1 1392],[1 0])
control_d = c2d(control, .001)
```

T

{zero-order hold is default