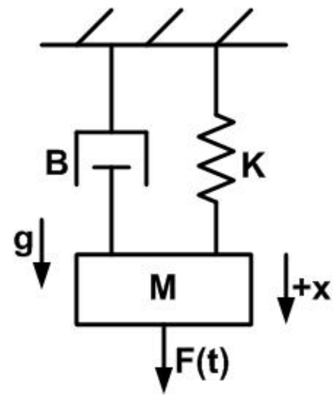
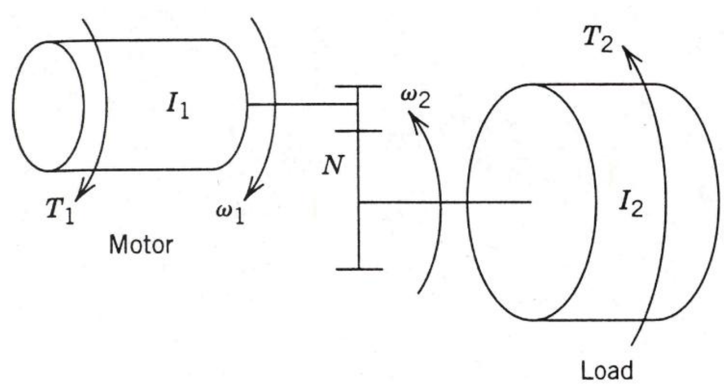
Electromechanical Engineering Systems
MEEN 2210 Spring 2011

Mechanical System Modeling Problems: 1-DOF Systems



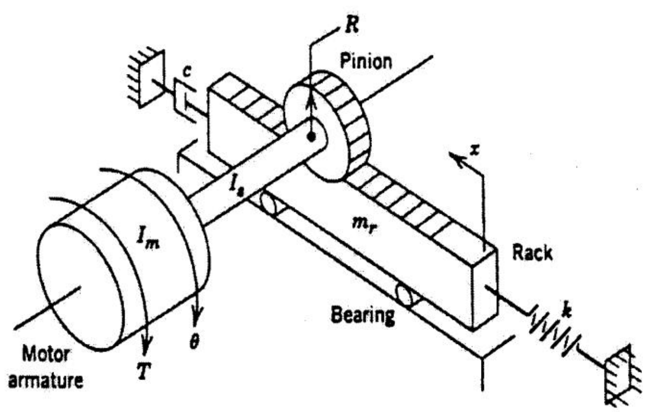
1. Consider the spring-mass-damper system shown. All mechanical systems have inertia, compliance, and energy dissipation.
   - State the simplifying assumptions that lead to this 1-DOF physical model.
   - Draw the FBD for the translating mass.
   - Derive the mathematical model, i.e., differential equation of motion, for this system. Write the transfer function x/F.
   - Show that if the absolute mass displacement x is measured from the static equilibrium position, as opposed to the unstretched spring position, the effect of gravity does not appear in the mathematical model. Explain why this is so.
   - Discuss the energy storage /dissipation characteristics of this model.
   - Put the 2^{nd}-order ODE in standard form. Relate the model parameters K, B, and M to the standard-form parameters ω_n, ζ, and K (steady-state gain).
   - Draw a block diagram that represents the mathematical model.
   - Represent the mathematical model in state-variable form.
   - Draw the electrical system analog for this mechanical system. Relate the mechanical system force f, velocity v, viscous damper B, spring K, and mass M to the electrical system voltage e, current i, resistor R, capacitor C, and inductor L.

2. Proper selection of the gear ratio can maximize the load acceleration for a given motor and load. For the spur-gear transmission system shown, consider the shafts and gears to be ideal (i.e., frictionless, massless, rigid) and N is the gear ratio ω_1 / ω_2.

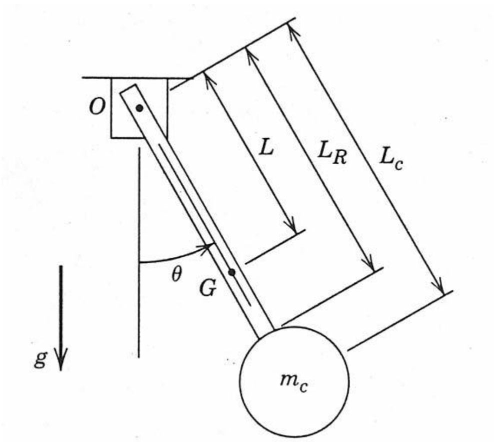


- Derive the mathematical model for this system. Write the equation of motion first in terms of ω_1 and then in terms of ω_2.
- Derive an expression for N that maximizes the load angular acceleration α_2.
- Set the load torque T_2 equal to zero and simplify the derived expression. State in words the result obtained.

3. Develop the equivalent rotational and translational models of the rack-and-pinion gear system shown. The applied torque T is the input variable, and the angular displacement θ or linear displacement x is the output variable. Neglect any compliance in the shaft. Bearings are frictionless. Rack-and-pinion gear is ideal. The pinion gear mass moment of inertia about its CG (geometric center) is I_p.

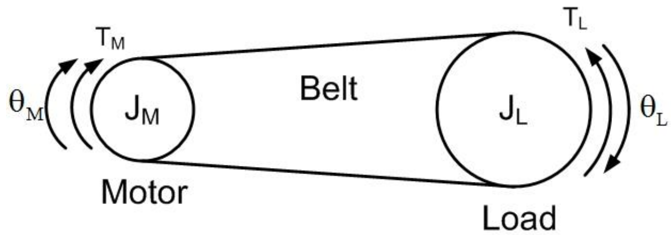


4. The pendulum shown consists of a concentrated mass attached to one end of a rigid rod. The rod is free to rotate at the other end about the frictionless pivot point O. The pendulum point mass m_C is a distance L_C from pivot point O. The rigid rod has a mass m_R and is homogeneous of length L_R with mass moment of inertia I_{RG} about its center of mass. The center of mass of the system (point mass + rod) is at point G a distance L from point O.

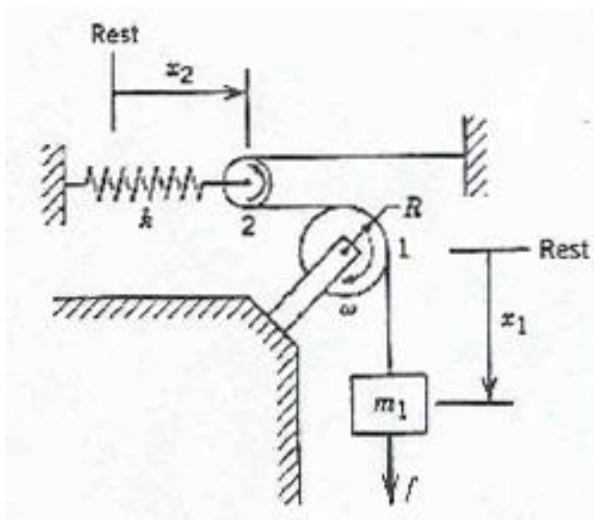


   - Determine the equation of motion for this system. This equation is nonlinear. Why? How can we linearize the equation of motion? Why should we linearize the equation of motion?
   - Discuss the case where the rod's mass is small compared to the concentrated mass.

5. For the belt-driven system shown, J_M and J_L are motor and load inertias, respectively, including pulley inertias. R_M and R_L are the pulley radii and B_M and B_L are the viscous damping coefficients for the motor and load, respectively. Treat the belt as rigid, i.e., neglect any belt compliance. Derive the equation of motion for the system first in terms of θ_M and then in terms of θ_L. Show the equivalence of this system to the system in problem #2 by comparing the equations of motion and transfer functions.



6. In the pulley system shown, assume that the cable is massless and inextensible. The input is the applied force f and the output is the displacement x_1. The pulleys are frictionless and there is no slip between the cable and the pulleys. (a) Assume the pulley masses are negligible and derive the system's equation of motion; (b) suppose the mass of pulley 2 is negligible, but pulley 1 has a mass m_p and a moment of inertia about its center of mass I_p; derive the system's equation of motion.



7. Determine the mathematical model for each of the levered systems shown with force f as the input. Assume small displacements. Neglect pivot friction. The center of gravity of the lever is at the pivot. In figure (a) the lever is rigid and massless; in figure (b) the lever is rigid and has a moment of inertia I relative to the pivot.