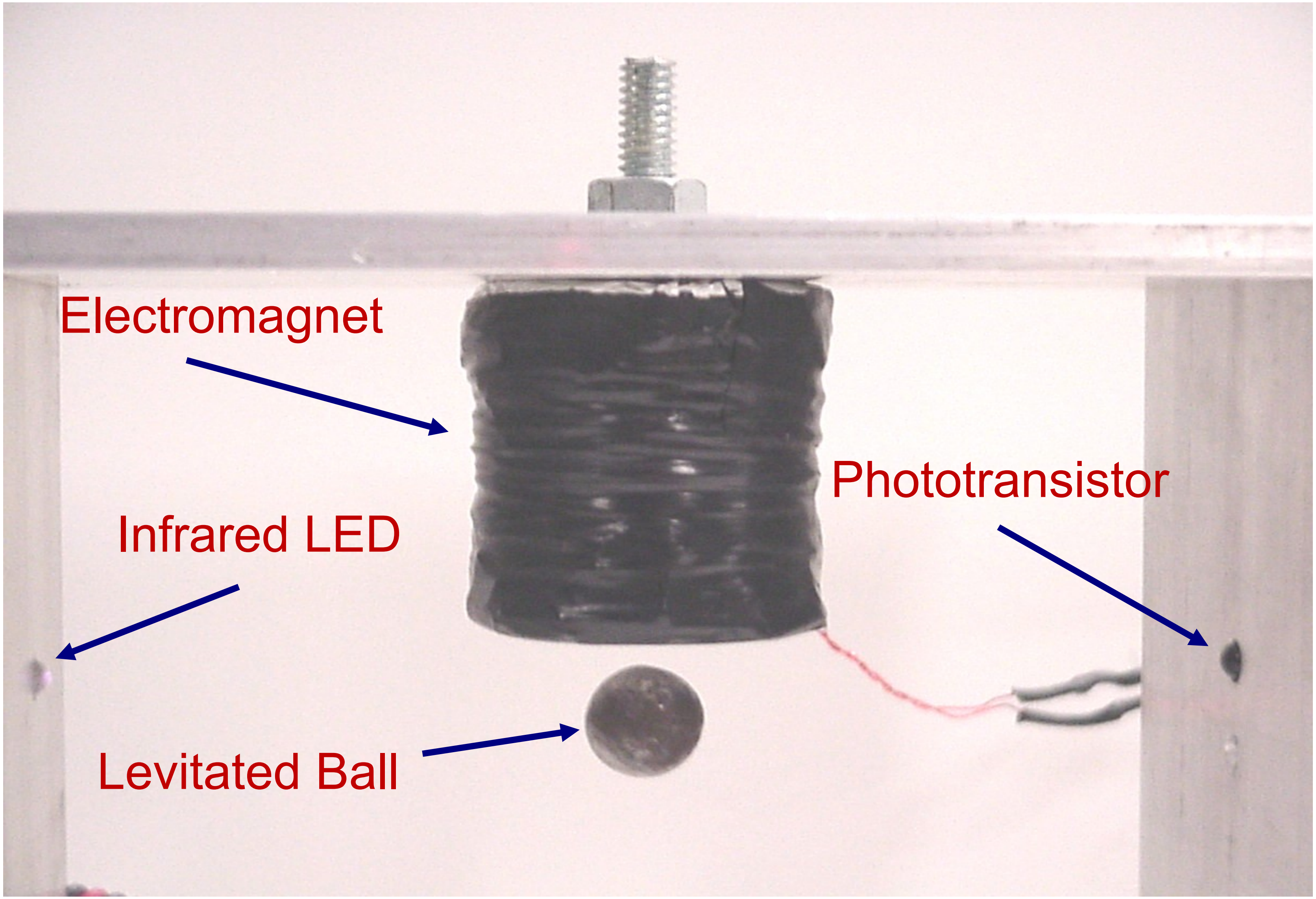
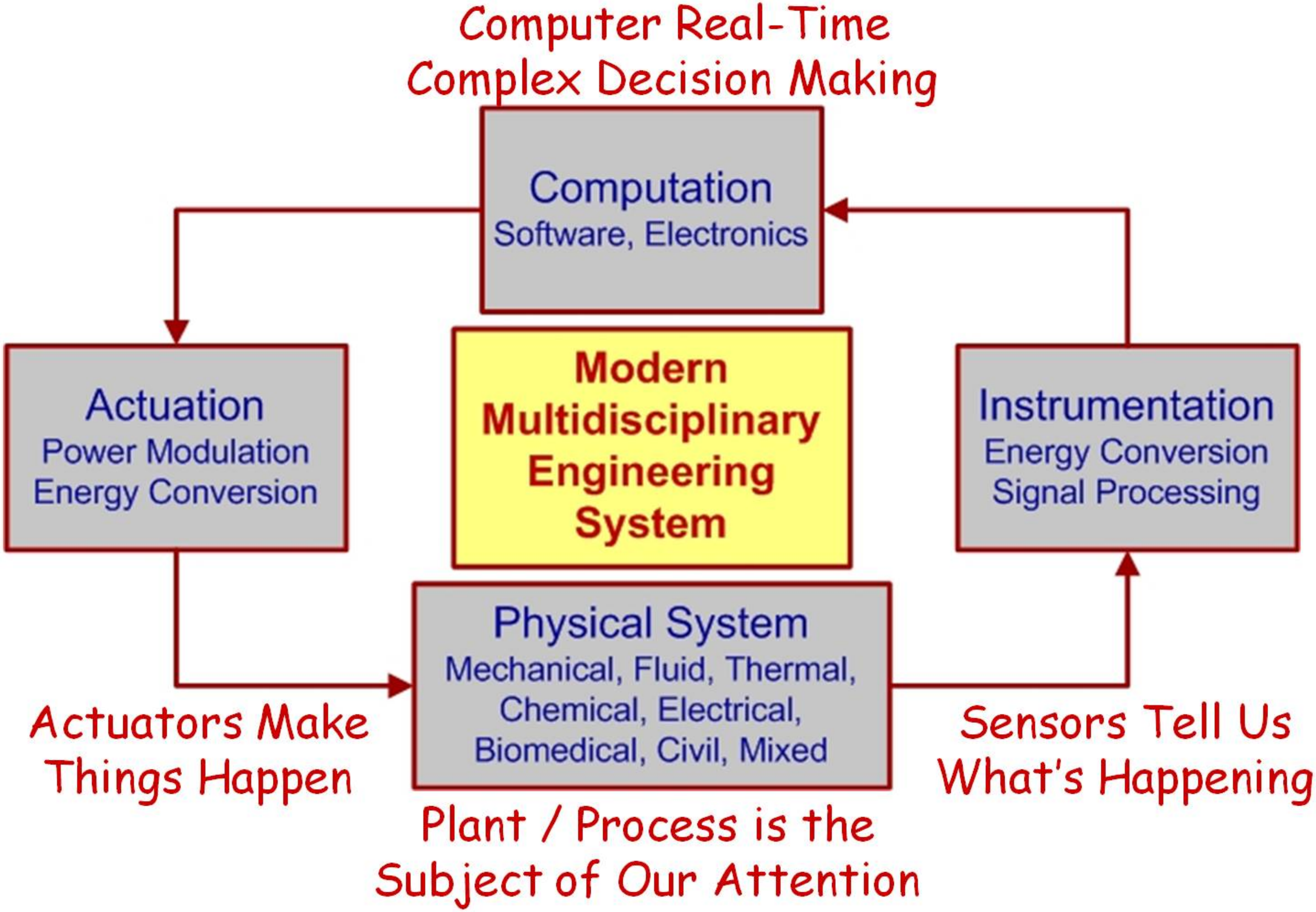
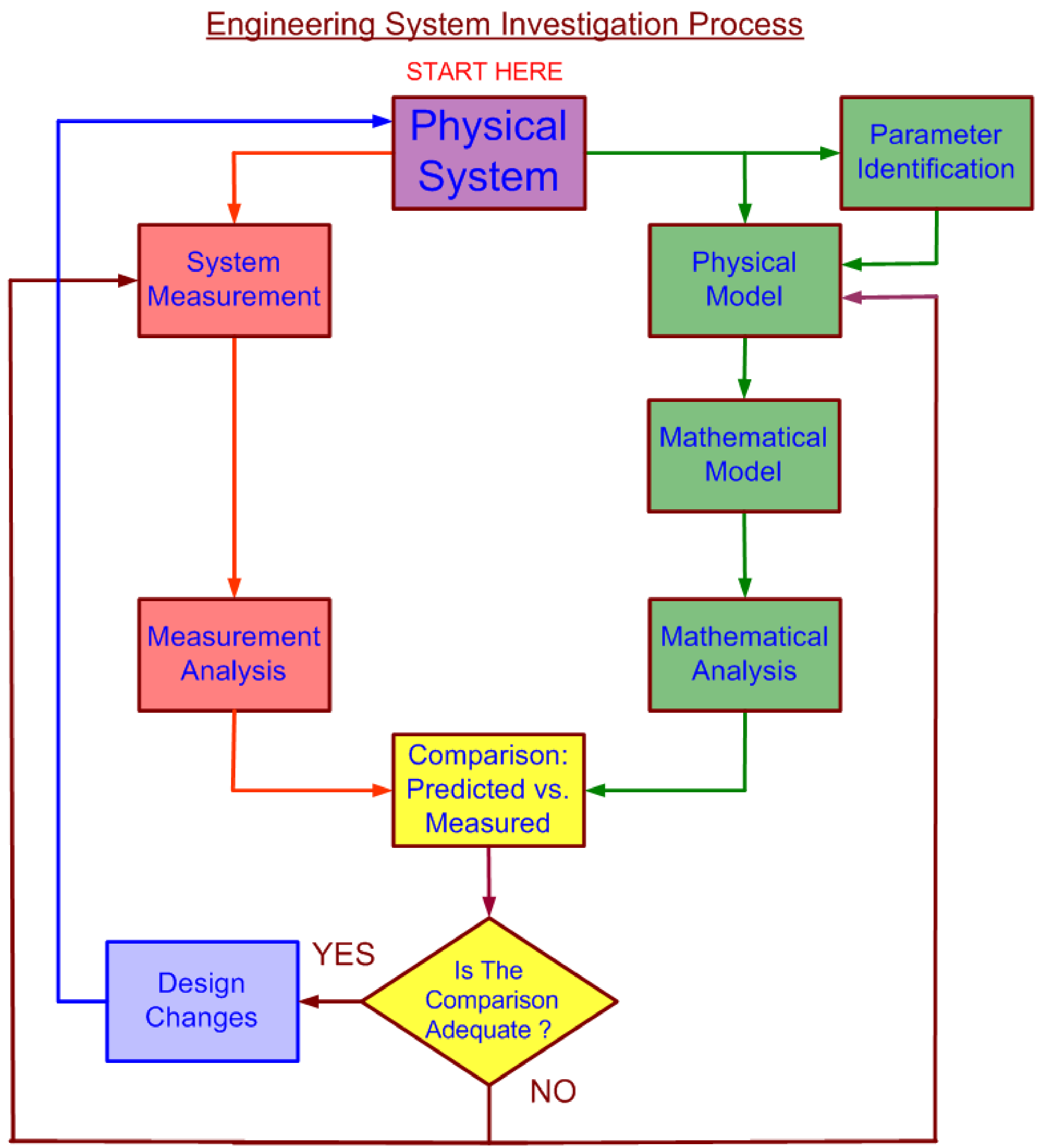
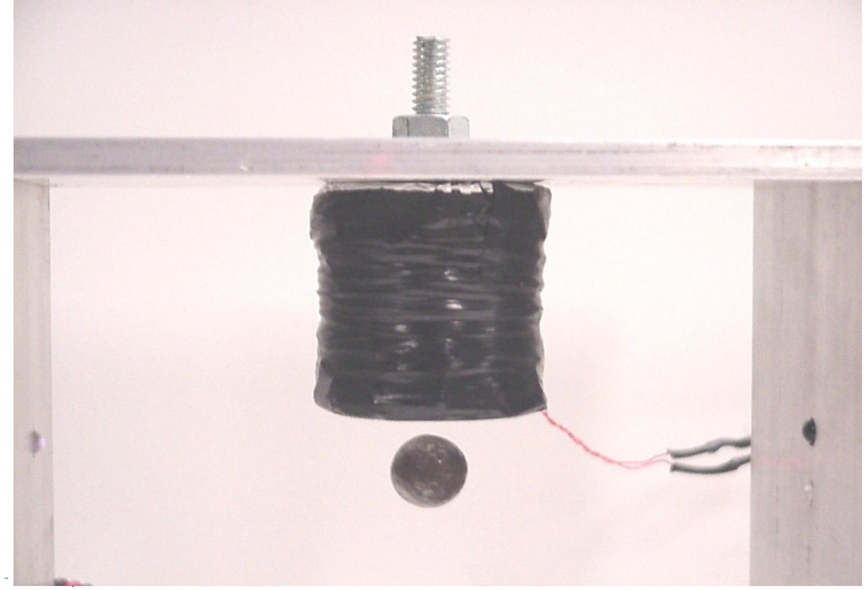
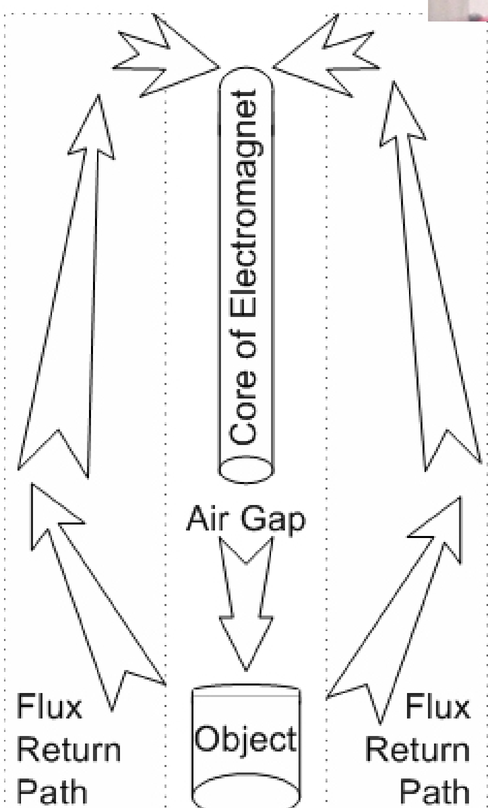
Magnetic Levitation System

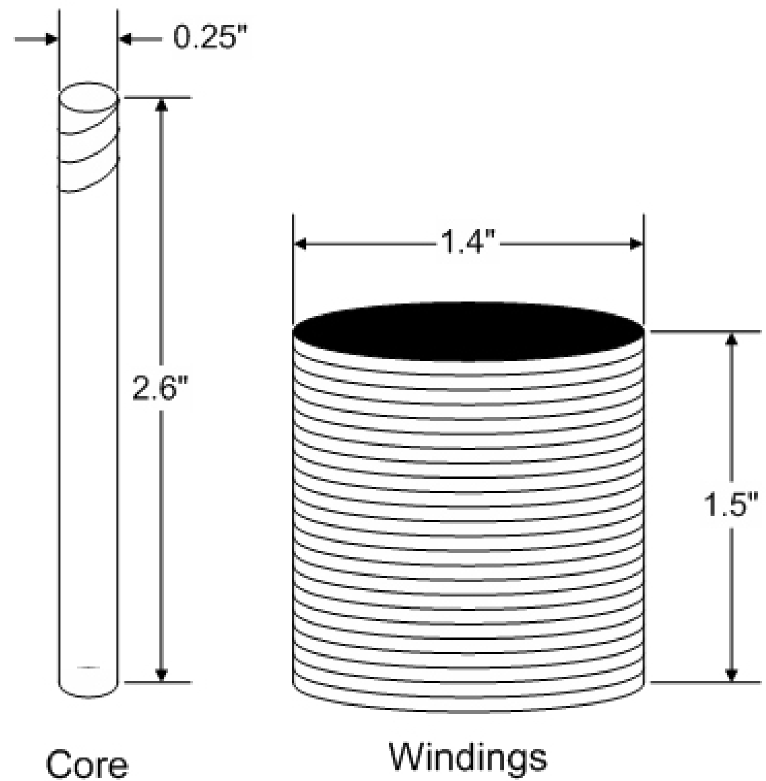


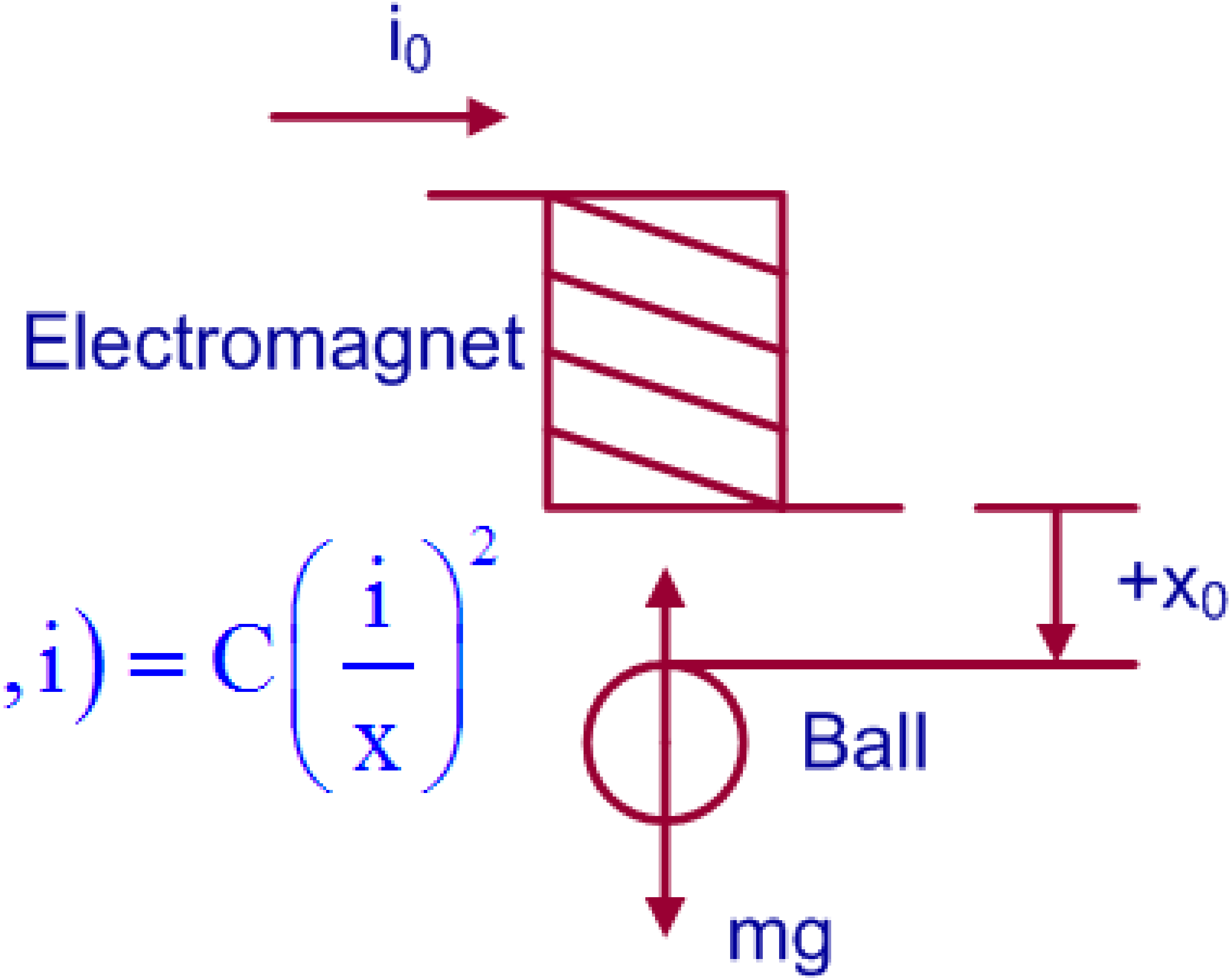

Computer Real-Time
Complex Decision Making
Computation
Software, Electronics
Actuation
Power Modulation
Energy Conversion
Modern
Multidisciplinary
Engineering
System
Instrumentation
Energy Conversion
Signal Processing
Physical System
Mechanical, Fluid, Thermal,
Chemical, Electrical,
Biomedical, Civil, Mixed
Actuators Make
Things Happen
Sensors Tell Us
What's Happening
Plant / Process is the
Subject of Our Attention


Engineering System Investigation Process
START HERE
Physical System
Parameter Identification
System Measurement
Physical Model
Mathematical Model
Measurement Analysis
Mathematical Analysis
Comparison: Predicted vs. Measured
Design Changes
YES
Is The Comparison Adequate ?
NO




Core of Electromagnet
Air Gap
Object
Flux
Return
Path
Flux
Return
Path


0.25"
2.6"
Core
1.4"
1.5"
Windings



$$f(x,i) = C\left(\frac{i}{x}\right)^2$$

Electrical / Magnetic Circuit Analogy

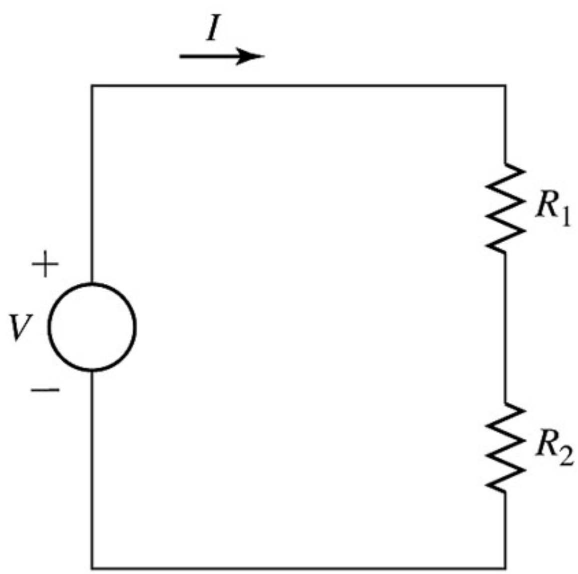
$$V = iR \qquad \Im = \Phi \Re \qquad \left\{ \begin{array}{l} V \Leftrightarrow \Im \\ i \Leftrightarrow \Phi \\ R \Leftrightarrow \Re \end{array} \right.$$

$\Im = Ni$ magnetomotive force (At)

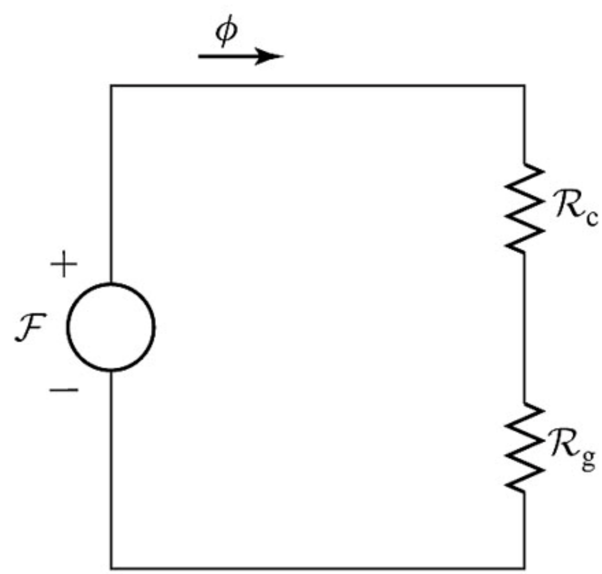
$\Re = \dfrac{\ell_c}{\mu A}$ reluctance (At/Wb)

$\mu =$ permeability

$$\left(R = \frac{\ell}{\sigma A} \right)$$



$$I = \frac{V}{(R_1 + R_2)}$$



$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

Magnetic Levitation System Derivation

$$\phi = \phi_\ell + \phi_m \quad \text{Neglect } \phi_\ell \quad \phi_m = \frac{Ni}{\Re_m}$$

$$\boxed{f(x,i) = C\left(\frac{i^2}{x^2}\right)}$$

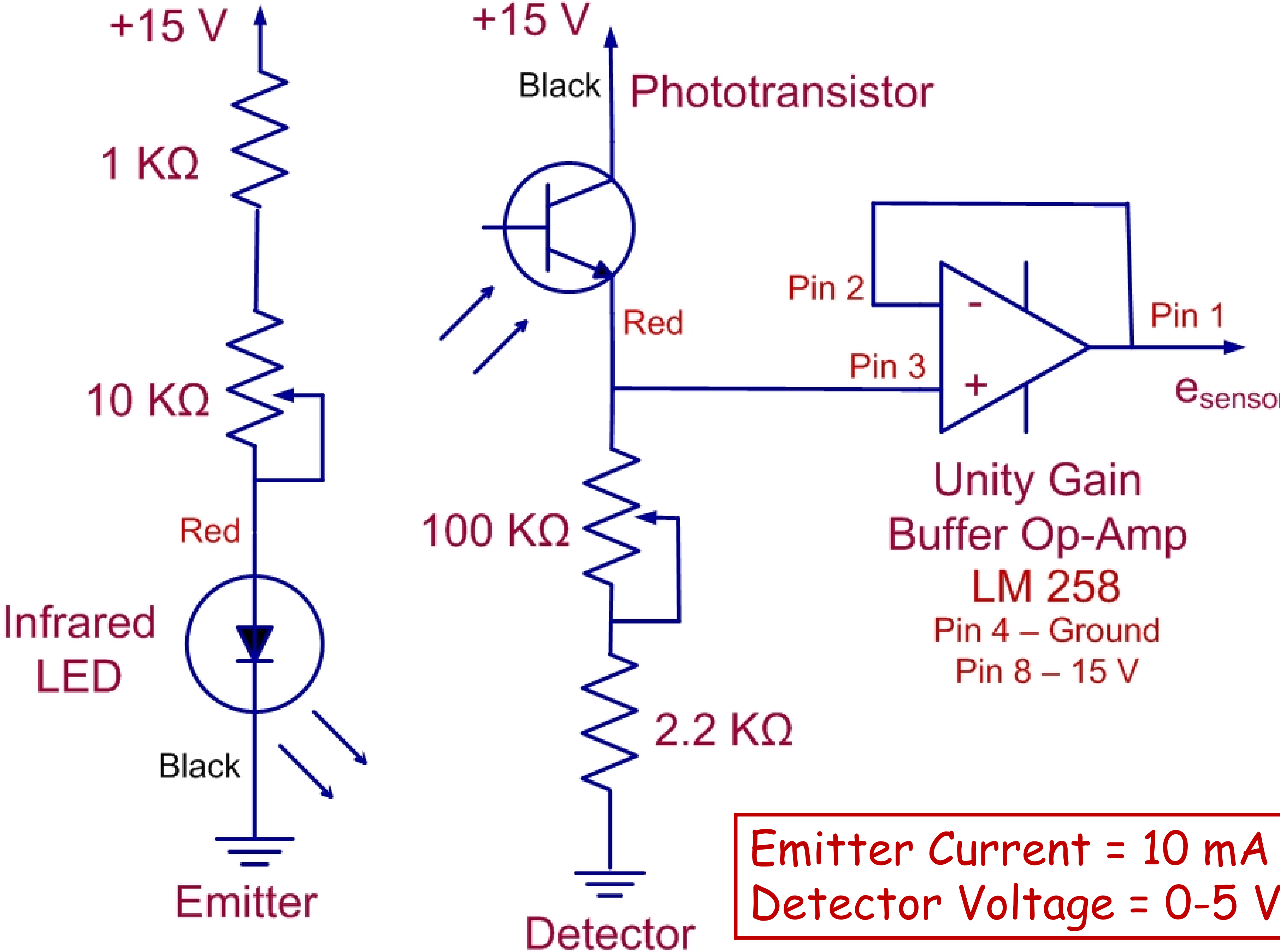
$$\lambda = N\phi = N\phi_m = \frac{N^2 i}{\Re_m} = L_m i \qquad \Re_m = \Re_{core} + \Re_{gap} + \Re_{object} + \Re_{return\ path}$$

Define: $\Re = \Re_{core} + \Re_{object} + \Re_{return\ path} = \text{constant}$

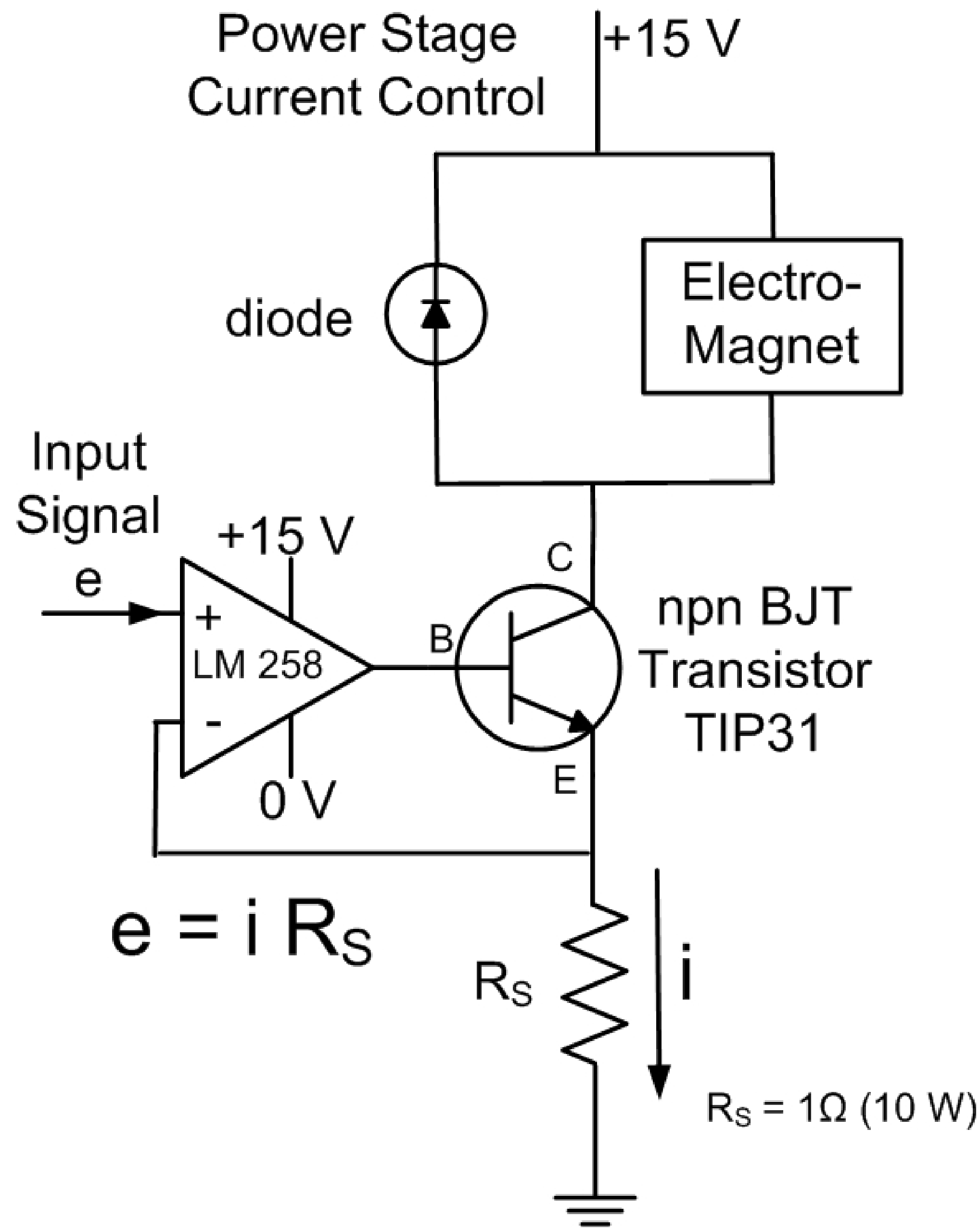
$$\Re_{gap} = \frac{x_{gap}}{\mu_0 A_{gap}} \qquad L_m = \frac{N^2}{\Re_m} = \frac{N^2}{\Re + \frac{x_{gap}}{\mu_0 A_{gap}}} = \frac{\mu_0 A_{gap} N^2}{\mu_0 A_{gap}\Re + x_{gap}}$$

$$W_{field} = \frac{1}{2} L(x) i^2 = \frac{1}{2}\frac{\mu_0 A_{gap} N^2}{\mu_0 A_{gap}\Re + x_{gap}} i^2$$

$$f_e = \frac{1}{2} i^2 \frac{dL(x)}{dx} = -\frac{1}{2}\mu_0 A_{gap} N^2 \left(\frac{1}{\mu_0 A_{gap}\Re + x_{gap}}\right)^2 = -K_1\left(\frac{i}{K_2 + x_{gap}}\right)^2$$

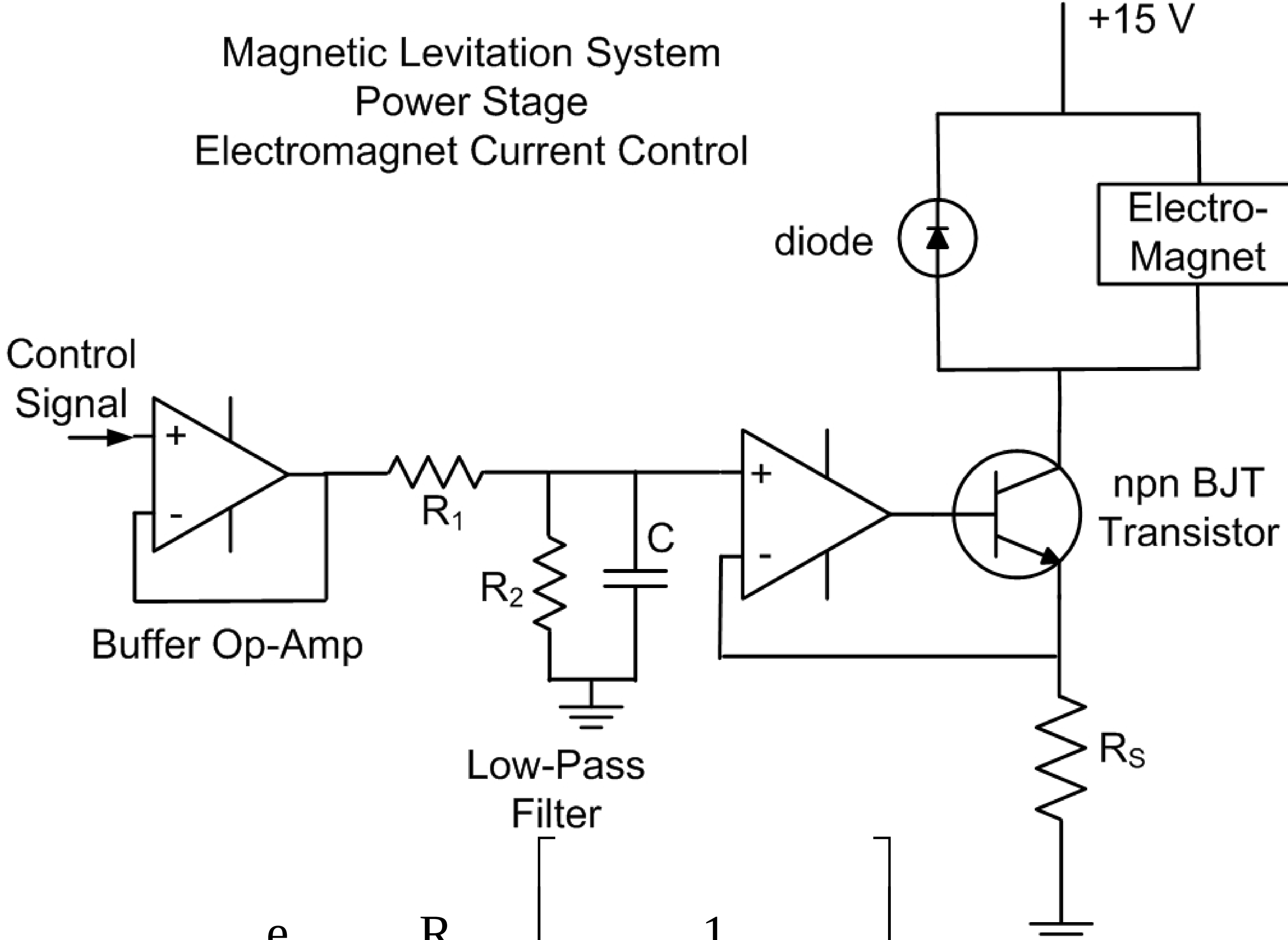


Emitter Current = 10 mA
Detector Voltage = 0-5 V

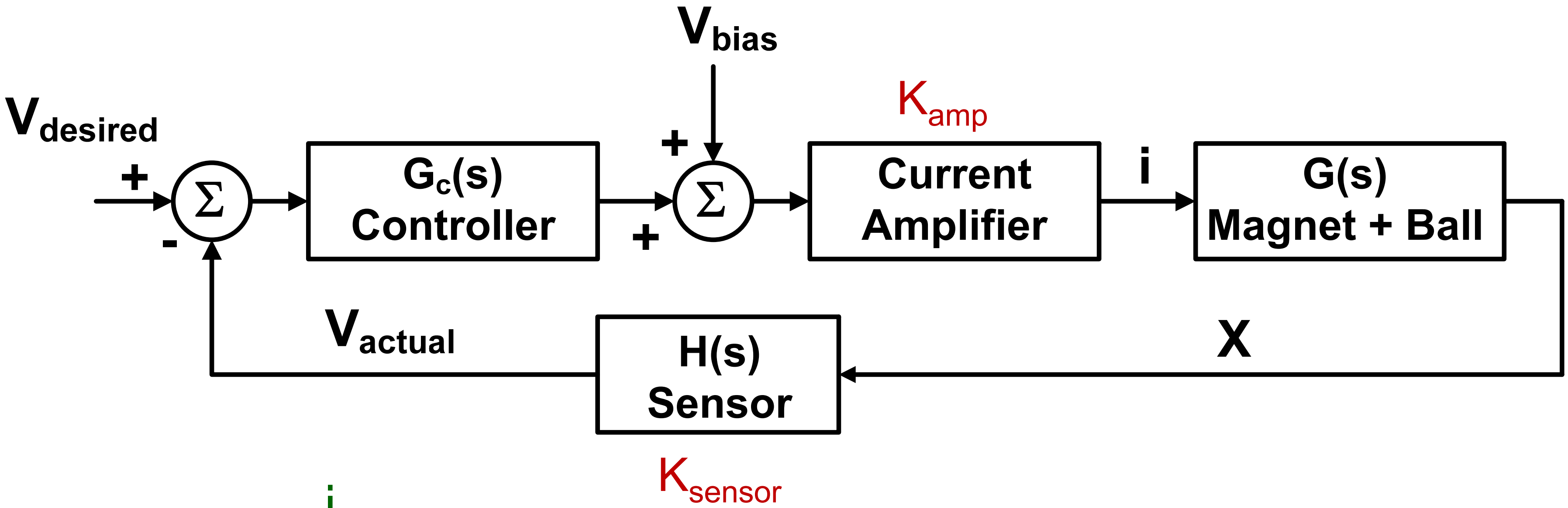


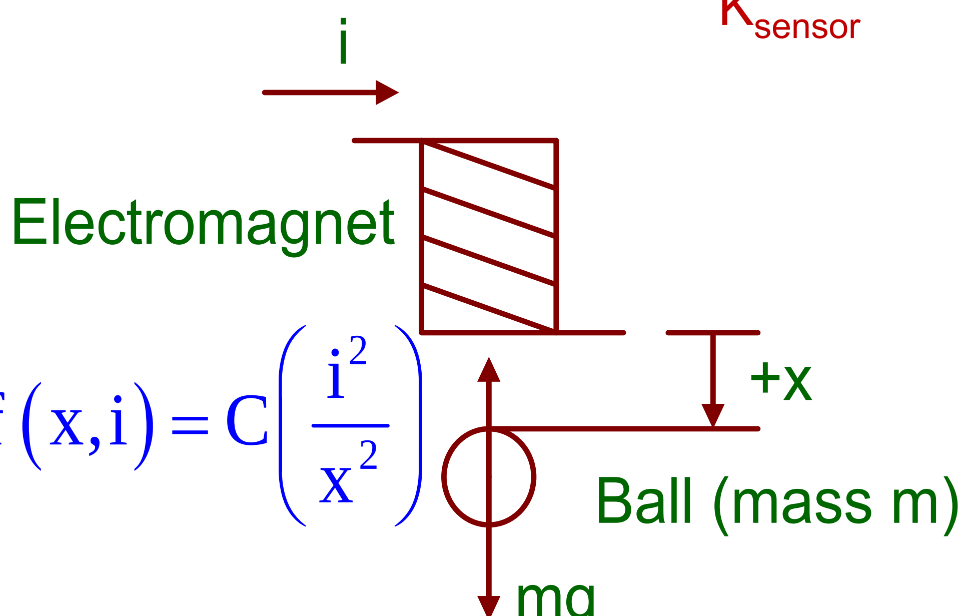
Power Stage
Current Control
+15 V
diode
Electro-
Magnet
Input
Signal
e
+15 V
+
LM 258
-
0 V
B
C
E
npn BJT
Transistor
TIP31
e = i RS
RS
i
RS = 1Ω (10 W)

Magnetic Levitation System
Power Stage
Electromagnet Current Control



$$\frac{e_o}{e_i} = \frac{R_2}{R_1 + R_2}\left[\frac{1}{\frac{R_1 R_2}{R_1 + R_2} CD + 1}\right]$$





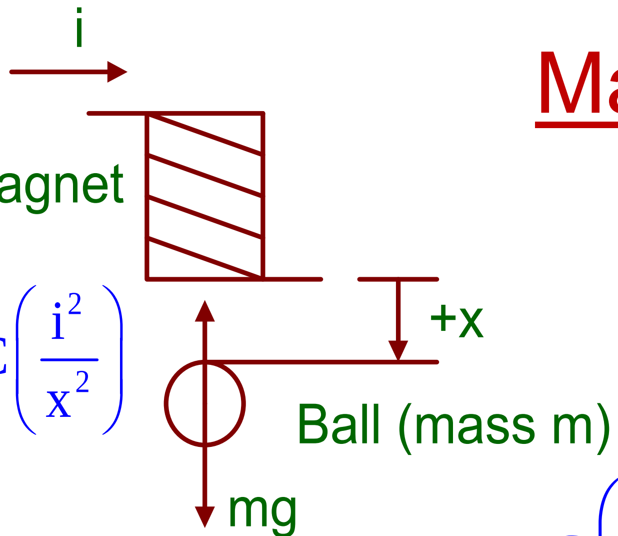
$$f(x,i) = C\left(\frac{i^2}{x^2}\right)$$

From Equilibrium:

As i ↑, x↓, & V_{sensor} ↓

As i ↓, x ↑, & V_{sensor} ↑

Magnetic Levitation System Control System Design



$$f(x,i) = C\left(\frac{i^2}{x^2}\right)$$

Equation of Motion:

$$m\ddot{x} = mg - C\left(\frac{i^2}{x^2}\right)$$

At Equilibrium:

$$mg = C\left(\frac{\bar{i}^2}{\bar{x}^2}\right)$$

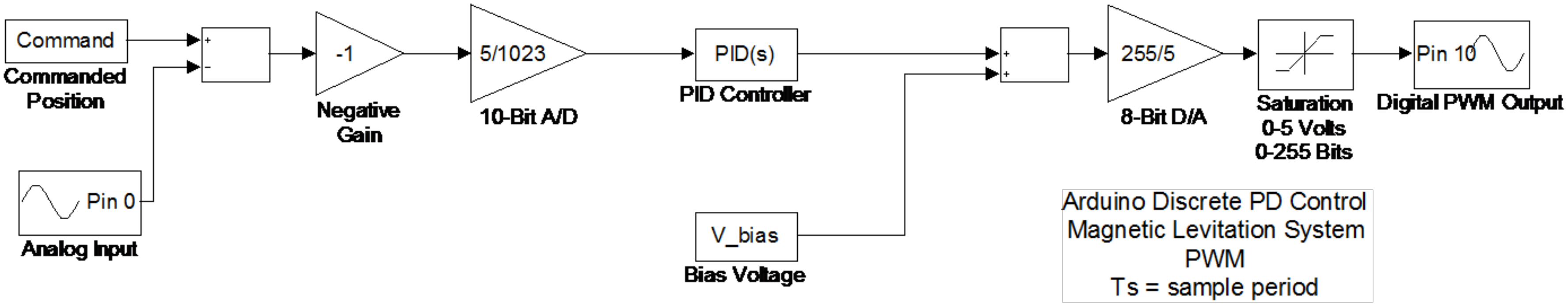
Linearization:

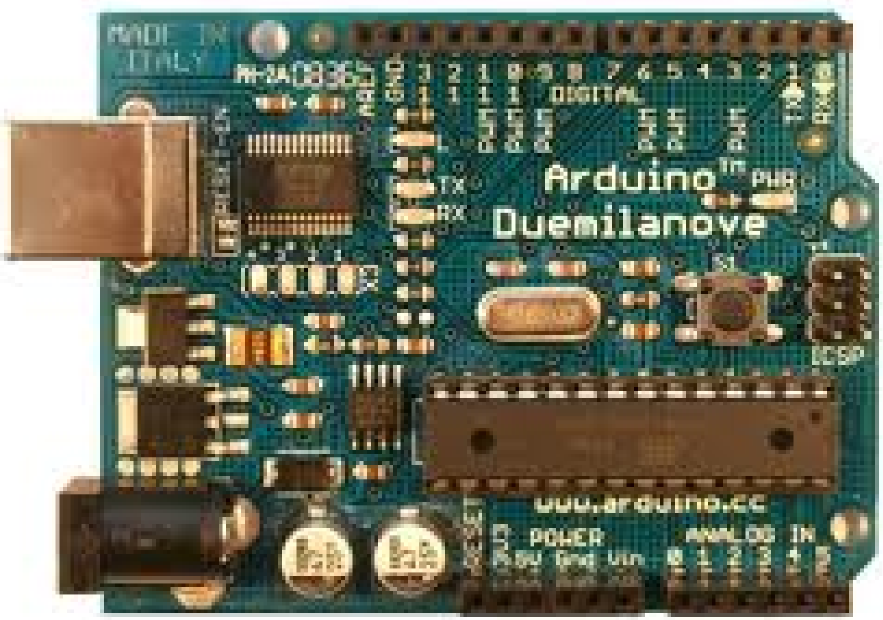
$$C\left(\frac{i^2}{x^2}\right) \approx C\left(\frac{\bar{i}^2}{\bar{x}^2}\right) - C\left(\frac{2\bar{i}^2}{\bar{x}^3}\right)\hat{x} + C\left(\frac{2\bar{i}}{\bar{x}^2}\right)\hat{i}$$

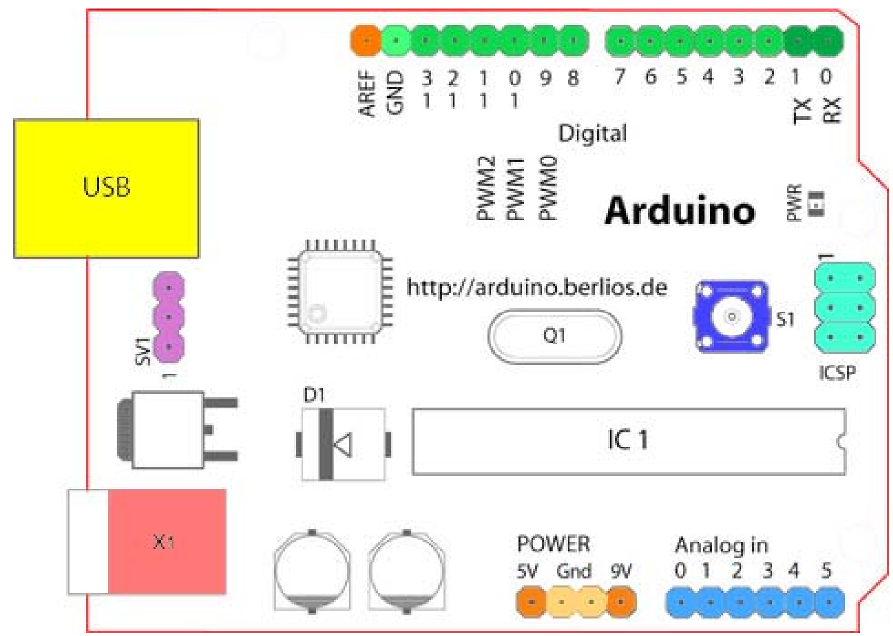
$$m\ddot{\hat{x}} = mg - C\left(\frac{\bar{i}^2}{\bar{x}^2}\right) + C\left(\frac{2\bar{i}^2}{\bar{x}^3}\right)\hat{x} - C\left(\frac{2\bar{i}}{\bar{x}^2}\right)\hat{i}$$

$$m\ddot{\hat{x}} = C\left(\frac{2\bar{i}^2}{\bar{x}^3}\right)\hat{x} - C\left(\frac{2\bar{i}}{\bar{x}^2}\right)\hat{i}$$

$$\frac{\hat{x}}{\hat{i}} = \frac{-K_1}{\left(s^2 - K_2\right)}$$


Command
Commanded Position
Pin 0
Analog Input
-1
Negative Gain
5/1023
10-Bit A/D
PID(s)
PID Controller
V_bias
Bias Voltage
255/5
8-Bit D/A
Saturation 0-5 Volts 0-255 Bits
Pin 10
Digital PWM Output
Arduino Discrete PD Control
Magnetic Levitation System
PWM
Ts = sample period


MADE IN ITALY
DIGITAL
Arduino
Duemilanove
ICSP
www.arduino.cc
POWER
ANALOG IN


AREF
GND
13 12 11 10 9 8
7 6 5 4 3 2 1 0
TX RX
Digital
PWM2 PWM1 PWM0
Arduino
PWR
USB
SV1
http://arduino.berlios.de
Q1
S1
ICSP
D1
IC 1
X1
POWER
5V Gnd 9V
Analog in
0 1 2 3 4 5

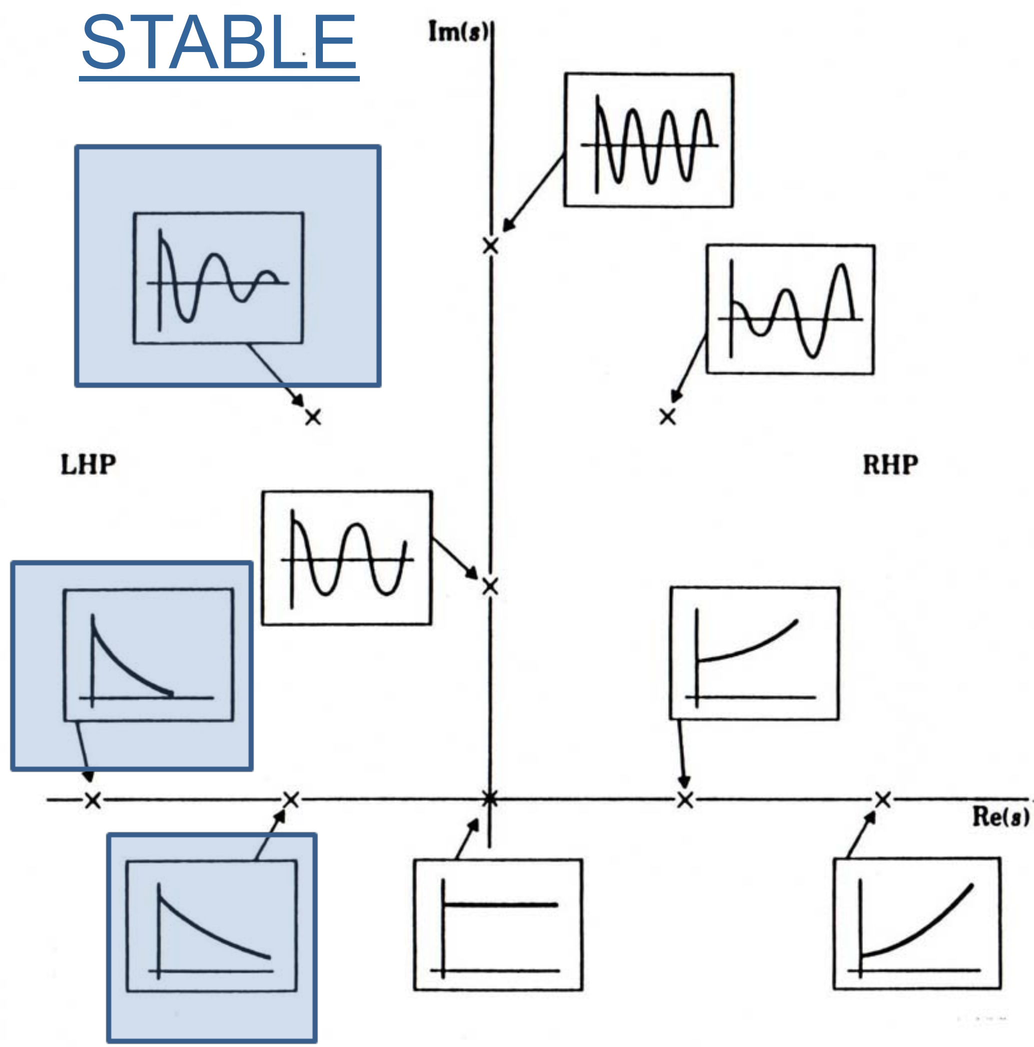
Absolute Stability

- If a system in equilibrium is momentarily excited by command and/or disturbance inputs and those inputs are then removed, the system must return to equilibrium if it is to be called absolutely stable.
- If action persists indefinitely after excitation is removed, the system is judged absolutely unstable.
- The analytical study of stability becomes a study of the stability of the solutions of the closed-loop system's differential equations.
- A complete and general stability theory is based on the locations in the complex plane of the *roots of the closed-loop system characteristic equation*, stable systems having all of their roots in the LHP.

Characteristic Equation & Stability:

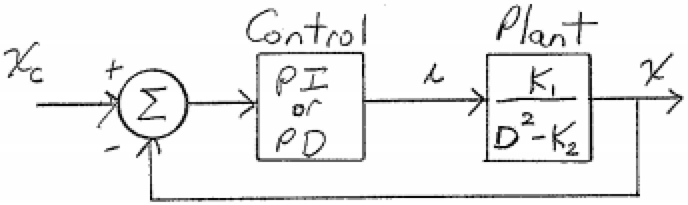
For absolute stability, all roots of the characteristic equation must have negative real parts, i.e., they must lie in the left-half plane, as shown. Note the following:

- If the system characteristic equation itself shows any sign changes, the system is always be unstable.
- If there are any gaps (zero coefficients) in the characteristic equation, the system is always unstable.
- Note, however, that a lack of gaps or sign changes is a necessary but not a sufficient condition for stability.



Time Functions Associated with Points in the Complex Plane

Magnetic Levitation System Control Design



PI: $K_p + \dfrac{K_I}{D}$

PD: $K_p + K_D D$

PI Control: Closed-Loop System TF

$$\frac{x}{x_c} = \frac{\left(\frac{K_p D + K_I}{D}\right)\left(\frac{K_1}{D^2-K_2}\right)}{1+\left(\frac{K_p D + K_I}{D}\right)\left(\frac{K_1}{D^2-K_2}\right)} = \frac{K_1 (K_p D + K_I)}{(D^2-K_2)(D) + (K_p D + K_I)(K_1)}$$

Closed-Loop System Characteristic Equation:

$$D^3 + (-K_2 + K_1 K_p) D^1 + K_I K_1 D^0 = 0$$

D^2 term missing! Unstable!

PD Control: Closed-Loop System TF

$$\frac{x}{x_c} = \frac{(K_p + K_D D)\left(\frac{K_1}{D^2-K_2}\right)}{1 + (K_p + K_D D)\left(\frac{K_1}{D^2-K_2}\right)} = \frac{K_1 (K_p + K_D D)}{(D^2 - K_2) + K_1 (K_p + K_D D)}$$

Closed-Loop System Characteristic Equation:

$$D^2 + K_1 K_D D^1 + (K_1 K_p - K_2) D^0 = 0$$

$K_1 K_p - K_2 > 0$ for stability!

$$D^2 + K_1 K_D D + (K_1 K_p - K_2) \Longrightarrow D^2 + 2\zeta\omega_n D + \omega_n^2$$

$$t_r, t_s, M_p \Longrightarrow \zeta, \omega_n \Longrightarrow K_p, K_D$$

Control Design